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Franklin A. Lindsay is now President of Itek Corporation and before that he was in a managerial position at McKinsey & Company.

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New Techniques for Management Decision Making

Franklin A. Lindsay

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8 July 1963
Franklin A. Lindsay
"New Techniques for
Management Decision Making"

PREFACE TO PAPERBACK EDITION

Since this report was written, five years ago, much has been done to further develop and apply analytical techniques to the solution of complex management problems.

One significant advance has been the development of PERT techniques for the cost and schedule control of complex development programs, such as the Polaris missile and submarine system. By means of these new analytical tools, slippages in schedules and cost overruns can be spotted before they would normally be apparent and corrective action can be taken. A tremendous amount of literature is available on PERT. An article in the June 1963 issue of *News Front* provides a general and nontechnical description of PERT and the types of problems that can be solved with it.

Other important contributions to the field have been made by men such as Jay W. Forrester (*Industrial Dynamics*, 1961, M.I.T. Press) and others who have developed computer models of complex and dynamic industrial production and distribution systems.

Since this study was written substantial progress also has been made in computer languages and programs, so that today the costs of preparing complex management problems for computer solution are considerably less than they would have been a few years ago.

In this edition I have substituted for the original example of a linear programming problem, a much better and clearer example which has been prepared by John B. Friauf.

Since the original publication of this report my own work has shifted from the field of management consulting to the ranks of industrial management. As a consequence, I now feel even more strongly that management must itself have some basic understanding of the capabilities and limitations of mathematical analysis as applied to management problems before these techniques can be used effectively.

PREFACE

Since World War II a variety of new analytic techniques for solving complex management problems have been developed from the disciplines of science and mathematics. The simultaneous development of high-speed computers has provided added means, and incentive, to mathematically and scientifically trained people to try their hand at some of the tougher problems of management. These techniques have been grouped under such titles as: operations research, systems analysis, probability theory, game theory, input-output analysis, and operational gaming.

The technical publications describing this work have grown apace. Unfortunately, much of the material has been written in technical language that is difficult for a nonmathematician to understand. Even the titles can be discouraging. For example: "Direct Use of Extremal Principles in Solving Certain Optimizing Problems Involving Inequalities," or "The Distribution of the Time Required to Reduce to Some Preassigned Level a Single-Channel Queue Characterized by a Time-Dependent Poisson-Distributed Arrival and a General Class of Holding Times," or "Gradient Methods for Constrained Maxima."

This report is designed as a bridge between the specialists and the managers who must have some understanding of the capabilities and limitations of these new decision-making tools if they are to be used intelligently. The report is divided into four parts:

1. An outline of the different ways these tools can be used, and an explanation of the central function of the mathematical model.
2. A description of the various analytical techniques, such as linear programing, which are available for solving management problems.
3. An examination of the actual and potential applications of these techniques to practical management problems.
4. An evaluation of the capabilities and limitations of these tools and of the factors that are key to their successful use.

The report is written for those who have had little or no mathematical training. In a few cases, some mathematical description has been included, but the reader can easily skip these short sections if he so desires.

FALL, 1958

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Part I.

AN INTRODUCTION TO MATHEMATICAL ANALYSIS

A chief executive often faces the problem of selecting from among several alternatives a plan that will commit his company to a major expenditure. He may, for example, have to choose a plan for expanding capacity in order to supply rapidly increasing markets. In doing so, he must consider many basic factors that will affect the profitability of the plan chosen, including: the future growth rate of the market, the future geographical distribution of the market, future price levels, production costs, labor costs, raw material costs, his company's share of the market, etc. Each of these involves a large element of uncertainty. But, since the executive cannot avoid making decisions, he and his assistants make the best estimates they can for each.

Unfortunately, the use of only a single estimate for each factor, such as the size of the market three years hence, can result in failure to consider some of the most significant situations that may arise in the future. If, to take a grossly oversimplified example, there are only ten independent factors to be considered, and if there is a two-thirds chance of each estimate's being correct to within 5 per cent (a high probability), the probability that all ten will be correct to within 5 per cent is only 1.7 per cent. If, instead, the probability of each of the ten factors' being correct to within 5 per cent drops to a 50-50 chance, the probability that all ten will be within 5 per cent range drops to less than 0.1 per cent. Thus the executive who used a single estimate for each factor would be ignoring other possible conclusions having a combined probability of occurrence of 99.9 per cent.

Clearly, to be certain that all the important combinations of future events are considered, management must study the implications of a very large number of alternative situations. Yet normal planning techniques can at best handle only a small number of combinations of specific projections of key factors.

Mathematical analysis gives the executive many new tools for improving his decisions: (a) by increasing the number of alternatives that he can consider; (b) by speeding up the decision processes; (c) by helping him to evaluate the relative risks of bold and conservative courses; and (d) by helping him to bring into optimum balance the many diverse elements of a modern enterprise.

MANAGEMENT'S NEED TO UNDERSTAND THE NEW TOOLS

Mathematical analysis is a tool to be used by management—not a substitute for it. Like anything else, it can be used improperly and can, in the wrong hands, become a very dangerous device. To a greater degree than heretofore, the professional specialist in the form of the mathematical analyst is moving directly next door to the executive offices of the corporation. If the executive is to gain the benefits of mathematical analysis, yet protect himself against its misuse, he must gain some understanding of both its capabilities and its limitations. The use of analysts in this way is fundamentally different from the use of technical experts for advice in their own fields—such as metallurgy or chemistry.

The relationship between the executive and the specialist is much more intimate in this new field than it is in the traditional technical fields. To a very considerable degree, management judgment must be combined with the technical skill of the specialist in selecting the problem, in defining the relationships among the various factors, and in interpreting the results.

WAYS OF USING THE NEW TOOLS

The new mathematical analytic tools for decision making can be used in several different ways:

1. To find the optimum combination of a very large number of interrelated factors.
2. To find the best solution, or at least a good solution, to problems involving uncertainty of future events.
3. To find meaningful relationships among many interacting factors and to separate the important factors from the unimportant.
4. To develop ways of collecting and processing significant data at minimum cost.
5. To relate quantitatively the operational objectives of the individual segments of a business or government organization, or even of individual workers, to the over-all objectives of that organization.
6. To test the sensitivity of any given solution to changes in controlling factors.

Most practical problems encountered will include several, if not all, of these uses of mathematical analysis. They are examined in greater detail below.

1. *Finding the optimum combination of a very large number of variables:* In many situations, all the factors are known and measurable, and their relationships are fully understood. In such situations, the only difficulty in finding the best combination is the very large number of alternatives that must be calculated and compared. Take, for example, a deceptively simple problem. A salesman wants to lay out a trip to all the state capitals, starting from and returning to New York, so as to minimize the distances traveled. The problem can be attacked in four different ways:

(a) The salesman can select his route by laying out a few alternatives that look "reasonable," measuring each of them, and selecting the shortest. This method will undoubtedly give a good quick solution; but it cannot, except by luck, find the best of all possible alternatives.

(b) He can, at least theoretically, lay out all possible routes and measure each in turn. But even in this simple problem, the number of possible routes is astronomical. It is approximately 10^{50}, or "one" followed by fifty zeros. If the salesman had started his analysis when the world was still cooling and before even one-cell life had appeared, and if he had measured

100,000 routes a second ever since, with no time off, he would by now just be getting started on the analysis! Obviously, many practical problems are of even greater complexity.

(c) He can use a new mathematical procedure worked out at the RAND Corporation, by which this problem was actually solved in seven steps, and which yielded the provably best route.

(d) Finally, he can lay out a few routes, just as in the first approach, and then by mathematical means estimate the probability that the best answer among these deviates from the true best answer by not more than a given percentage. If the probability of a greater deviation is higher than he is prepared to accept, he can then try a few more routes with the objective of reducing the chance of an unacceptably large error.

The third and fourth approaches are typical of the uses of mathematics in handling complexity. In this case, the use of either of these latter methods, instead of the first approach, probably would be justified only if something more costly than a single salesman were involved. But in many other problems, the potential payoff of even a 1 per cent improvement is so high that the cost of the analysis will be repaid many times.

It should be recognized, of course, that many problems of a similar or greater complexity still cannot be solved precisely, and a large portion of these may never be susceptible of a precise solution at a reasonable cost.

2. *Finding the solution to problems involving uncertainty:* Many management problems involve decisions based on future events that cannot be precisely predicted. Mathematical analysis provides a way of describing and evaluating uncertainty which is known as "probability theory." Some types of uncertainty can be precisely described in mathematical terms derived from simple assumptions of probability; other types of uncertainty can only be described as mathematical approximations; and for some problems, human intuition is probably still the best means of evaluation. For example, the probability of rainfall, and therefore of other events dependent upon it, can be expressed mathematically quite accurately. The probability of new customers' switching from a competitor as a result of sales calls or advertising cannot be expressed in precise mathematical terms, but can be estimated from past performance. The probability of a

strike, on the other hand, can often be best estimated by the intuitive judgments of mediators, labor reporters, and other experts.

But regardless of how an estimate of the probability of the occurrence of a future event is made, it can usually be used in the solution of a management problem, provided the implications of probable error in the estimate are correctly weighed.

3. *Finding meaningful relationships among many interacting factors:* In many actual situations, it often appears impossible to sort out and identify those factors that, together, control the desired result. Even if it were possible to vary only one factor at a time, the information thus obtained would probably be of little value in predicting the combined effects of all factors simultaneously. Under such circumstances, management has been forced to act on fragmentary and piecemeal data, which at best give poor guides to future action. This is particularly true of marketing problems because so few of the significant factors are under the control of the seller.

For example, how much of a 75 per cent increase in the volume of beer sold through a given distributor is attributable to local advertising, to national advertising, to sales promotion, and to the effort and skill of the distributor? How much is attributable to the corresponding activities of competitors? How much to price? Finally, how is this increase related to the share of the market at the beginning of the period measured? If the answers to these questions were known, the brewer would know more accurately how much money to allocate to advertising and sales promotion and what distributor markup and price concessions to allow.

The major analytical problem is to find, not only the effect attributable to each factor separately, but also the effect attribuable to complex interactions of price, advertising, and sales effort. For a given advertising expenditure would have one result with a low point-of-sales effort, and a very different result with a higher point-of-sales effort.

So far, the tools of mathematical analysis have been applied to such problems on an experimental basis only. But it can be said that these tools now offer a means of analyzing past data on interacting factors and of designing further experiments that could produce useful conclusions with a minimum of data-collecting effort and with a minimum of costly interference with the system being studied.

4. *Collecting and analyzing data at minimum cost:* The use of mathematical analysis in decision making inevitably leads to increased requirements for data. Since much of the information needed is seldom readily available from company records, data collection becomes an almost unavoidable preliminary problem. Data collection in most cases represents the bulk of the costs of mathematical analysis, and it is important to find a means to do it economically.

Sampling techniques (discussed at some length in Part II) are one mathematical tool that is proving to be increasingly useful for data collection. Much work has been done on sampling in the last few years; and, as a result, several alternative sampling procedures are available for most data-collection problems. Each procedure has its own associated costs, but it is possible to minimize the total cost of sampling required to gain an acceptable degree of accuracy. For example, one may have a choice between, on one hand, a sophisticated technique that requires only a small sample to yield an answer within a given expected error, and, on the other hand, a simpler technique that requires a larger sample.

No amount of analysis can breathe significance into a mass of statistics unless the required information is actually contained in the data. But a good statistical analyst can often, through better anlalysis, uncover significant information that a handbook statistician would either miss or misinterpret.

5. *Relating operational objectives of parts of an organization to the organization's over-all objectives:* If a problem is to be solved by mathematical means, it is necessary to define the objective so that it can be measured quantitatively and, if there are several elements of the objective, to relate them all precisely so that the over-all objective can be measured in numerical terms. It is in failure to take this latter step that some of the greatest dangers of mathematical analysis lie. To cite several examples: Reduction in idle machine time as an objective may result in greater in-process inventories. The reduction in air-freight costs for emergency parts delivery may result in greater losses in the assembly department. And a drive to cut bad-debt losses may result in credit limitations that depress sales—and profits—by more than the amount of the loss avoided.

The selection of a limited goal for a subordinate part of the total activities of a corporation, and the organization of this limited activity to realize the limited goal, is called "partial optimization" or "suboptimization." While it is essential that such limited goals be set for subordinate units, great care must be observed to be sure that the goals selected are consistent with the over-all objective of the corporation.

In many problems, objectives of different departments may be in conflict. The treasurer might consider the objective of inventory policy to be the minimization of capital tied up in inventory. The production department would regard as optimum an inventory policy that permitted the longest production runs and a stable level of operations. The sales department would regard as optimum an inventory policy that always permitted immediate deliveries from inventory. If such a problem were to be analyzed mathematically—in order to determine reorder points and quantities that would minimize *total costs*—it would be necessary to reconcile these conflicting objectives into a single measure representing the over-all profit interests of the company.

An example of a problem "suboptimized" with an incorrect objective is a World War II operations research study which showed that the larger a convoy of ships, the smaller would be the losses resulting from enemy submarine action. Although this conclusion proved to be correct, the solution failed to meet the real objective, which was maximum delivery of supplies to Europe. As the size of convoys was increased, a point was reached beyond which the over-all deliveries were actually reduced; for the increased delays of loaded ships waiting for convoys to assemble, and the increased delays in unloading in congested British ports, more than offset the reduction in sinkings. A study that, instead, took as it objective the maximization of tonnage delivered to Europe before December 1944, would have found the optimum convoy size that minimized the combined losses arising from loss of ships and from port delays.

A particular value of mathematical analysis is that it makes it extremely difficult to sweep under the rug such knotty problems on objectives. If one is studying the operation of an airline, it is not sufficient to say that the criterion for effective scheduling is to maximize the flying time of each aircraft and maximize the load factor or percentage of seats sold. To define mathematically the criteria for scheduling, one must show precisely how these two measures, as well as others, act on one another in their contribution to profits. For example, an added night flight may increase aircraft usage, but reduce the average usage of seats as well as increase maintenance problems. The real questions are: how are these and other factors related, and what is the real measure of good scheduling? Mathematical analysis provides a rigorous discipline which makes it hard to duck such questions.

6. *Testing the sensitivity of the solution to variations in individual factors:* Mathematical techniques provide a greater opportunity to explore a broad range of solutions and to estimate the sensitivity of the solution to changes in individual elements or in combinations of these elements. Fortunately, most management problems appear to be relatively insensitive to changes over a relatively wide range or "band."

If the manager knows the width of this band, he will know how precise must be his management controls to be sure he does not move out of his optimum range of operations. He will also know that, even though the precise optimum is at one point, he can safely move away from that point by a given margin in order to accommodate other factors, such as good will. He can also form some estimate of the value of improving some of his management controls. For example, it has been shown in one case that, if one has a good method of periodically correcting production levels for errors in forecasting, a mediocre forecasting technique is adequate and large expenditures to improve forecasting further probably will not be worth while.

In short, mathematical analysis can often show the penalties, if any, for operating under conditions that are theoretically less than optimum.

THE ROLE OF THE "MODEL"

A common technique for solving technical problems is to construct a "model" of a machine or an activity, which can be used to simulate its operation and thereby to predict its behavior in an actual situation. Models of aircraft are always tested in wind tunnels before production is started. Often the model is not a physical replica. For example, the stresses inside a rotating shaft connected to a turbine have been studied by constructing an electrical model in which the distribution of stresses under various loads can be measured by passing electrical currents through a liquid held in a container. Models of many types are being used increasingly to study complex processes.

In mathematical analysis, the word "model" is used to mean a mathematical description of an activity which expresses the relationships among the various elements with sufficient accuracy that it can be used to predict the actual outcome under any expected set of circumstances. These models are of many types and their characteristics depend on the real-life situation they simulate. The obvious advantage of a model is that it, instead of the organization it simulates, can be manipulated in a variety of ways until the best solution is found. The actual organization can then be operated in an optimum fashion with a minimum of costly experiments and adjustments.

The great disadvantage is that no model can completely duplicate reality; therefore both the construction of the model, and the translation of the results to the actual problem, must be done with both sophistication and humility.

The function of model building and analysis is not to replace intuition and judgment, but rather to support them with tools for handling complexity and uncertainty with which the human intuition cannot cope unaided. In a sense, the use of a model frees the intuition and permits it to concentrate on those problems to which it is particularly suited. It permits the creative manager to test rigorously the implications of new plans, new schemes, and new ideas.

Part II.

TECHNIQUES OF MATHEMATICAL ANALYSIS

This section describes some of the more important tools of mathematical analysis, such as linear programing, queuing theory, and input-output analysis. The breakdown of the subject into these headings runs the danger of misleading the reader to assume that actual problems will always fall neatly into one or another of these categories. Actually most solutions will incorporate several of these techniques. The approach should never be to try to find problems on which linear programing, for example, can be used. Instead, one should select the problems most important to management and then see what tools of analysis, as well as what combination of analysis and judgment, will give the best solution.

Some of these tools are of proven use in solving management problems; others have had limited application but appear to have considerable potential use; still others have only limited possibilities of practical application, at least at their present state of development. Techniques falling into the latter group have been included because they have been discussed extensively in both technical and general management literature.

These techniques are presented in three groups: probability techniques, which are used in decision making under conditions of uncertainty; programing techniques, which might be described as algebraic means of reaching rigorous mathematical solutions of problems involving the complex interaction of many variables; and simulation techniques, which are systematic trial-and-error means of solving problems and especially of finding the implications of different policies and alternative external events. It is important to recognize, however, that this is a somewhat arbitrary grouping. Some of the techniques under one heading are closely related to those under the other. Moreover, frequently a management problem is solved by a combination of techniques in different groups.

MATHEMATICAL APPROACHES TO DECISION MAKING UNDER UNCERTAINTY

Statistics and Probability

Modern statistics is an important tool for solving many management problems. A statistician is often thought of as one who is concerned solely with accumulating masses of "numbers" generated by other people and who records and summarizes these numbers in some sort of semiclerical fashion. In reality, a modern statistician can make fully as important a contribution to the success of an enterprise as can a physicist or engineer.

E. Bright Wilson, Professor of Chemistry at Harvard, defines the function of statistics as follows:

> It is the purpose of statistical analysis to provide methods of treating data so that the maximum information can be obtained with a predetermined risk of drawing false conclusions. No method of analysis can extract more information from a set of data than is contained therein, and no method, statistical or otherwise, can draw conclusions from experimental data with zero risk of error. The use of statistical methods is based on the reasonable assumption that accepted principles of logic and probability should produce correct answers more often than guessing.*

The Role of the Statistician

What should a manager expect from a statistician? A statistician can perform at least two essential functions: he can collect data in the most efficient manner, and he can draw useful conclusions from it. Data collection in the statistical sense usually involves sampling existing information as a means of reducing costs, and often will also involve the design of special tests and experiments to produce additional needed information that is not contained in reports and records collected in the normal course of business. In the field of data collection the statistician can:

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* E. Bright Wilson, Jr., *An Introduction to Scientific Research,* McGraw-Hill, 1952.

1. Set up sampling programs so that the maximum of useful data can be collected at minimum cost and effort.
2. Design testing programs involving the simultaneous interaction of many different factors, and indicate the way these factors should be varied in order to yield the most information.
3. Reduce or eliminate bias in his data by the way he designs the data-collection programs and by careful testing of the data collected.

Through data analysis, the statistician can establish the nature and magnitude of relationships that may be suspected to exist among the various factors affecting the outcome of an actual situation under study. More specifically, he can perform these functions:

1. Although he will never be able to give categorical conclusions, he will always be able to state the degree of confidence one can have in the answers he produces. For example, he may state that the conclusion drawn from analyzing a sample has a 95 per cent chance of being within 10 per cent of the real answer, or a 90 per cent chance of being within 5 per cent of the real answer.
2. He will be able to reduce to any desired level the risk of drawing false conclusions (but only at the cost of collecting more information and of more intensive analysis of that data).
3. He will often be able to predict the probability of each of several possible outcomes for a future event.
4. He will be able to conclude whether a relationship probably exists between two factors or whether an apparent relationship is more probably due simply to chance.
5. He will be able to sort out the relationships among many different factors, all of which bear on the outcome of an event or activity, and to determine both the effect that each individual factor has on the outcome and the combined effect of the interaction of any two, three, or more of the factors.
6. Perhaps most important, he will be able to prevent needless errors in the anlysis of data which an untrained person will unwittingly commit.

In performing these services, the statistician makes use of numerous techniques, three of which—sampling, probability theory, and factorial analysis—are discussed here.

Sampling

In some situations sampling provides the only possible means of drawing conclusions about a series of events. In others, it provides the most economical means of reaching conclusions.

A. *Necessity of sampling:* The necessity of sampling can be seen in these examples of management problems:

1. If blasting caps were to be 100 per cent test-fired, one would know exactly what percentage were duds but would have none left to be sold.

2. In deciding on the feasibility of a particular dam site, knowledge of the annual water runoff is critical, but if one waits until floods occur to gain this knowledge, it will obviously be too late to build a dam. Hence one must "sample" the runoff that has occurred in past years to predict the probable runoff during the life of the dam.

3. It would be prohibitively expensive for a manufacturer of breakfast food to ask every consumer in the United States to try a new product before deciding whether or not to launch the product. The use of test programs, or "samples," is thus the only feasible way of estimating customer acceptance in advance.

In fact, sometimes sampling will produce a more accurate result than will a complete count. In one experiment, six people were asked to count the number of "e's" appearing on two successive pages of a book. Their results were: 577, 495, 600, 558, 577, 569. Next, an estimate was made by counting every tenth line. The estimate based on the sample was 611 and took about one-third the time required for a full count. The actual number of "e's" on the two pages was 622. Thus the estimate made from the sample was closer to the truth than any of the full counts. The reader can easily repeat the experiment. (If the population census of the United States were made by sampling instead of by complete count, the results probably would be no less accurate than a full count and the cost would be far less.)

B. *Methods of sampling:* The basic question in sample design is an economic one. The more accurate the conclusions drawn from a sample, the more it will cost. Increased accuracy can be obtained in two ways: by increasing the sample size and by using more powerful means of analysis. The former increases the cost of data collection; the latter usually means spending more money on the design and analysis of the sample. The skilled statistician will have the judgment necessary to find the optimum plan.

Before a sampling program can be set up, it is necessary to determine the degree of acceptable error in the conclusion to be drawn. The acceptable error, for example, in estimating potential customer reaction to a new product might be greater than the acceptable error in estimating accounts receivable for auditing, and less than the acceptable error in estimating the total number of bolts in a stockroom bin. In some cases the optimum sampling error can be calculated as that which will minimize the combined costs of sampling and added costs of operation caused by the probable error.

For any given problem, there are usually several alternative means of sampling available. The principal types are:

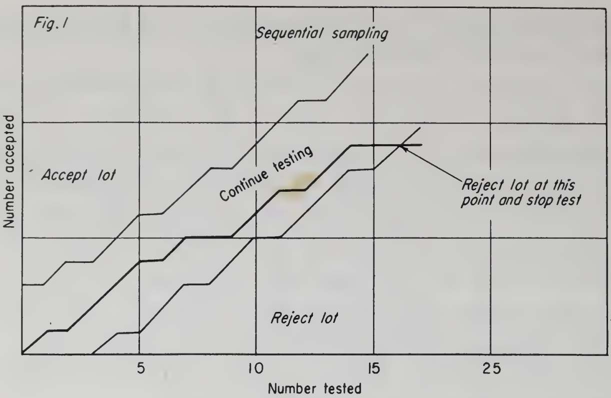
1. "Random" sampling, in which the items to be sampled are selected without plan.

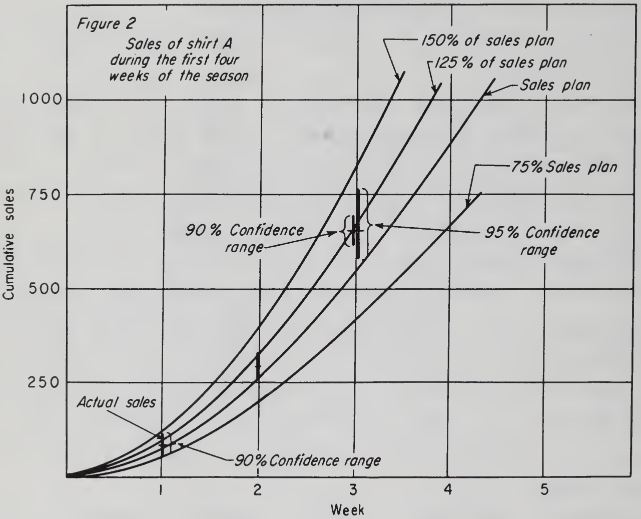
2. "Systematic" sampling, in which the items to be sampled are determined by taking, for example, every tenth one. This procedure is satisfactory as long as it does not produce biased results. Sampling the supermarket sales of soap on every seventh day would obviously produce a bias as it would select the same day each week.

3. "Stratified" sampling, in which the samples are selected in fixed proportions from specific subgroups. For example, an estimate of total department store receipts might be made by drawing separate samples from sales of under $5, $5 to $10, and over $10. The accuracy of estimates can normally be increased by use of stratified samples. Furthermore, one can be assured that the estimate will not, under most circumstances, vary more than a specified amount from the actual value.

4. "Sequential" sampling, in which a series of samples is taken until the results are sufficiently conclusive to justify discontinuing the process. Sequential sampling may make possible a reduction of as much as 50 per cent in the number of items sampled and is very useful where sampling costs are high, as in the testing of electronic equipment.

For such a test, a sample is initially selected from the production lot to be tested. The testing is started, and the cumulative record of acceptances and rejections is plotted as shown in Figure 1. The chart is divided into three areas: a center area represents the results that are not conclusive either way; the area on the left-hand side represents acceptance; and the area on the right represents rejection. The





tests are continued until the line of cumulative results crosses into one or the other of the two areas. The lines separating the three areas are determined so as to minimize the combined costs of the sampling process and of the probable error of the conclusion.

A modification of sequential sampling can give a manufacturer useful guides so that he can revise his production schedules as early as possible. For example, a large manufacturer of men's shirts needs to know, as quickly as possible after he has introduced a new line, which style will sell the best, which will be moderately profitable, and which will not sell at all. To get this answer, he might use the method illustrated in Figure 2. There the total sales of "Shirt A" are projected at 10,000, and the production rate has been scheduled accordingly. As each successive report of weekly sales is received, a line is plotted representing the actual sales of "Shirt A" in relation to the planned schedule of sales.

The first week 85 shirts are reported sold. The company statistician will determine the degree of confidence that can be placed on this first report as a prediction of the total sales. Figure 2 shows both the actual sales reported and the 90 per cent confidence range. For this first report, there is thus a 90 per cent chance that the total sales for the season will run between 8,000 (80 per cent of plan) and 14,000 (140 per cent of plan). This would undoubtedly be interpreted as an indication that production should not be cut back but that it was too early to make a decision to increase production to a higher level.

At the end of the second week, a new report and its 90 per cent confidence range is plotted. It shows a 90 per cent chance that total sales will run between 10,000 and 13,000 shirts. Management may decide at this point to increase the production rate if inventories are low. If not, the decision can be put off until the third week. At the end of the third week, the 90 per cent confidence range is shown to have narrowed to between 11,800 and 13,000 shirts. At this point, management may ask its statistician for the confidence range for a 95 per cent probability. On being told that there is a 95 per cent probability that the total season's sales will ultimately be between 10,500 and 14,000 shirts, management decides to increase production to 12,000 shirts (a 20 per cent increase), reasoning that there is only a 5 per cent chance that total sales will run less than 11,800 and a 2.5 per cent chance that they will drop as low as 10,500, while there is a 50 per cent chance that at least 12,600 shirts will be sold. At each succeeding week, new estimates and new decisions on production rates will be made until production is completed.

Sequential sampling can be used in many other ways. A life insurance statistician, after analyzing past records of insurance sales trainees and their later performance, could establish a series of performance guides for the periodic weeding out of potentially unsuccessful men. For example, at the end of three months, a trainee with a performance less satisfactory than that shown at the end of three months by 90 per cent of past trainees might be predicted to have only a 25 per cent chance of being ultimately successful. Past data might reveal that at the end of six months, a trainee whose performance was poorer than 75 per cent of the trainees would have only a 10 per cent chance of success, while a trainee whose record was poorer than 90 per cent would have only a 2 per cent chance of success.

With this information, management could set the lower limit of performance to be used in dropping trainees, with accurate knowledge of the probable number of potentially successful people who would be lost to the company as a result. Management might thus decide that, at the end of three months, it would keep all trainees whose performance gave them a 30 per cent or better chance of success, while at the end of six months, it would keep only those who had a 60 per cent or better chance of success. Further, it would be possible to determine analytically the cutoff percentages at different periods that would minimize the probable combined losses resulting from (a) keeping trainees who later turned out unsatisfactorily and (b) dropping trainees who might ultimately have proved to be successful salesmen.

Probability and Decision Making

Most management decisions involve weighing the probabilities of different outcomes of future events that the manager can neither control nor predict with certainty. For example, a manager cannot control the price policy of his competitor, nor can he predict the future market size.

Actuaries in the insurance business have, for many years, employed probability theory in estimating risks for which they must quote insurance rates. In spite of this successful application, probability theory is very seldom used in making the vast majority of management decisions. One often hears a business executive, a diplomat, or a soldier advocate taking a "calculated risk"; yet 99 out of 100 of these men have never actually calculated the risks they are assuming. In many cases, it is, of course, impossible to make such calculations. But, in many others, reasonably accurate calculations of risks—and of the costs associated with these risks—can be made.

In some types of problems, experienced judgment is frequently surprisingly accurate when compared with rigorous mathematical calculations, but in the assessing of probabilities it is quite often unreliable. For example, ask your friends some evening how probable it is that, of 25 people in a room, at least 2 have the same birthday. I have tried this and obtained answers varying between odds of

"100 to 1" to odds of "10,000 to 1." Calculation of the true probability shows that there is actually a slightly better than even chance that at least 2 of the 25 will have the same birthday. If the group is increased to 60, the odds go up to over 99 per cent.

A. *Most probable development not always best basis for planning:* Management decisions are often made on the basis of a single forecast figure, such as sales or Gross National Product, which is projected into the future. In a loose way this projection is considered to indicate the most probable course of future events and therefore the best assumption on which to base decisions. If, however, the penalties for the estimate's being inaccurate in one direction are greater than those for inaccuracies of comparable degree on the other side, it probably will not be the best course to make decisions on the basis of the most probable development.

Assume, for example, that an airline wishes to determine how many flights to schedule between Boston and New York between noon and midnight on the Friday before Labor Day. Clearly it will be next to impossible to schedule exactly the number of seats that will be sold. There will always be either too many or too few, because of the random element in the demand and because of fixed aircraft capacity. The objective of management thus will be to maximize its profits under conditions of uncertainty.

The probability for each of several alternative estimates is given in Table 1. The airline statisticians might have developed such a probability table on the basis of such things as traffic records of similar periods in the past, weather probability, and the estimated growth trend in passenger traffic. The single most probable estimate is that there will be 500 passengers. The probability that the number will be between 476 and 525 is 20 per cent. The probability for each successive increment of 50 then drops off in both directions. Thus, if flights sufficient for only 325 passengers are scheduled, there is a 96 per cent chance that there will be more potential passengers than seats and a 4 per cent chance that there will be empty seats. The probable loss arising out of either an overage or a shortage of seats is the cost of such an occurrence multiplied by the probability that it will occur.

The airline estimates that the net return on each seat sold (ticket price less variable costs per seat of making the flight) will be $15; therefore the cost in lost revenue, from providing fewer seats than could be sold, will be $15 per seat. Similarly, the airline estimates

Table 1

Number of passengers	*Probability*
676-725	.04
626-675	.07
576-625	.12
526-575	.17
476-525	.20
426-475	.17
376-425	.12
326-375	.07
276-325	.04

that the out-of-pocket cost of each unsold seat is $5. There is a 4 per cent probability that the number of people wanting seats will be between 676 and 725 (a median of 700). If only 375 seats are scheduled, the probable cost would be: .04 (700-375) x $15 = $78.00.

Similarily, there is a 7 per cent probability that the number of potential passengers will be between 626 and 675, and the probable cost will be: .07 (650-375) x $15 = $288.75.

Similar calculations can be made for the other probability ranges. When the number of passengers falls below the number of seats provided, the cost becomes $5 per seat, the cost of an unsold seat. Thus, for the 7 per cent probability that the number of potential passengers will be between 326 and 375 (median 350), the probable cost of unsold seats becomes: .07 (375-350) x $15 = $8.75.

The total probable costs of all the alternative possibilities when 375 seats are flown is $1,968. Similar calculations can be made for different numbers of seats scheduled. The results are shown in Table 2.

If the airline schedules flights with 500 seats, the most probable demand, there will be a 49.8 per cent chance of empty seats and an equal chance of customers' being turned away. The expected reduction in revenue will be $805. If, instead, 575 seats are scheduled, there will be a 77 per cent probability of having empty seats on each flight and a 23 per cent probability of having to turn away passengers. Yet, because the cost of a lost sale is greater than the cost of an empty seat, 575 seats represent the "best answer"—for the minimum probable loss of $639.

Thus the most probable forecast is not the best on which to base

Table 2

Seats scheduled	Probable loss
325	$2,654
375	1,968
425	1,389
475	918
500	805
525	705
575	639
625	703
675	900

plans unless the costs and probabilities of falling short are exactly equal to the costs and probabilities of being above the forecast.

B. *Analysis vs. judgment in estimating probability:* Most management decisions involve not one but several chance factors, each interacting with the others in their influence on the outcome to be predicted. In some cases, it is possible to calculate fairly rigorously the combined effect of several random factors. To do this, one must be able to form some fairly reliable estimate of the relative probability of each alternative outcome for each separate factor. This can often be done by analyzing the behavior of the same factor in the past. Rainfall statistics of the past provide good data on which to base probability estimates of future rainfall.

In other cases, reasonably good probability estimates can be developed by relating the event being studied to another event for which the probability distribution is known. For example, estimates of sales

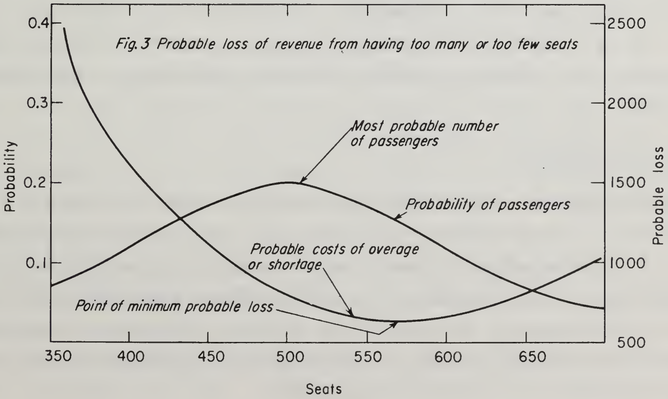


Fig. 3 Probable loss of revenue from having too many or too few seats

of air conditioners can be related to weather; and, in turn, the probabilities of various summer temperature patterns can be used to estimate the probabilities of air conditioner sales.

Sometimes, probabilities can be derived directly from theoretical knowledge of the behavior of random events. A statistician need not study records of past poker hands in order to determine the probability of drawing a specific poker hand.

In still other cases, informed judgment is the only possible way to make estimates of uncertain future events. For example, in assessing the likelihood of a strike in order to calculate the optimum size of reserve stocks, past records of strikes may be of no value. Instead, a probability estimate can be built up from the judgments of individuals, such as arbitrators, mediators, labor reporters, and company negotiators. Such a procedure may be criticized by theoretical statisticians; but practical decisions will, in any case, be made on somebody's estimate of the probability of a strike, and it is undoubtedly better to make this judgment explicit than to bury it in a guess at the size of a reserve stock that seems "about right."

C. *Total effect of several random factors:* After estimating the individual probabilities, it is necessary to combine these probabilities of individual events into over-all probability estimates of the outcome. If the individual probabilities can be described in simple mathematical terms, it is relatively simple to combine them into a single probability distribution. But if they do not conform to standard mathematical expressions, a much more laborious computation is required. If the problem is a large one, a technique known as "Monte Carlo" (discussed later) can be employed to approximate the answer by making repeated trials with randomly drawn inputs and recording specific outcomes. The probability of different dice throws can in this way be estimated by making a large number of throws and recording the relative number of times that each combination appears.

Factorial Analysis

In the solution of practical problems, one often has to separate the effects of many different variables. Only occasionally is it possible to hold constant all factors except one, while varying this remaining factor in order to isolate its particular effect on the solution. Even where this appears to be possible, it is often a dangerous procedure because some changes may occur unobserved and hence distort the apparent effect of the factor being studied.

Modern statistical analysis turns this problem around and succeeds in making a virtue out of the fault. By means of what has come to be known as "factorial design and analysis," it is possible to study the complex interactions of many variables in a way that will yield the maximum amount of significant information for the minimum experimental cost, with the assurance that all the conclusions are within any prescribed degree of probable accuracy.

Assume that a manufacturer of chemical products wishes to test the relative merits of three new products he has developed before deciding on the best one to market. There are three different industrial uses for the products and he wishes to test each product against each use. If he is limited to traditional techniques, he may decide to select three potential users and ask each to conduct three successive tests on each of the three products applied to *one* of the three uses. In so doing, he may reason that he is thus holding all variables but the products constant, and that he will, by averaging each group of the three tests, obtain a true measure of the relative value of each product for each use.

In actual fact, however, he is running a risk of seriously distorting his results because of variations: in the way different users handle each material, in the way a single user may handle each of the different products, and in the way each user measures his results.

But, by carefully designing his experiment to employ the technique of factorial analysis, the manufacturer not only can eliminate the effects of these differences among customers, but also, at the same time, can obtain additional useful information without increasing the number of tests required. Under some circumstances, he might actually reduce the number of tests required. The basis of factorial analysis is the use of a different combination of factors in each individual test so that no two experiments will be alike and so that all significant or possibly significant combinations of factors are tested.

In this case, the tests would be arranged so that each customer performed one test on each product combined with each use. The total number of tests, twenty-seven, would be the same as before; yet it would now be possible to isolate the effects of each variable, including:

1. The difference in over-all effectiveness of each product
2. The difference in effectiveness of each product for each particular use
3. The variation in over-all results due to differences in the way each user makes his tests

4. The relative sensitivity of each product to the differences in the way each user handles the product

A. *Isolating the effect of each variable:* The principle of factorial analysis can be demonstrated by an example in which it is desired to isolate the separate effects on a company's sales of the company's price, its advertising expenditure, the competitor's price, and his advertising expenditure. In order to make the example manageable, only two prices and two different levels of advertising are assumed to be possible. In this example, there are therefore sixteen different combinations of company and competitor policies, as shown on the table below.

Table 3

Sales Volume (1,000 units)

		Advertising			
Company (A)*		$10,000/ mo.	$10,000/ mo.	$20,000/ mo.	$20,000/ mo.
Competition (B)		$10,000/ mo.	$20,000/ mo.	$10,000/ mo.	$20,000/ mo.
Sales price					
Co. (C)	Comp. (D)				
$ 5	$ 5	105	90	140	120
5	10	180	160	220	200
10	5	60	50	80	70
10	10	95	80	120	115

* Letters in parentheses denote factors used in the mathematical example on page 25.

The average increase in company sales resulting from an increase in company advertising from $10,000 to $20,000 is:

$$1/8\ [\ (140\text{-}105) + (120\text{-}90) + (220\text{-}180) + (200\text{-}160) + (80\text{-}60) + (70\text{-}50) + (120\text{-}95) + (115\text{-}80)\ ] \times 1{,}000 = 30{,}625 \text{ units}$$

Similar effects on company sales caused by an increase in the competitor's advertising, an increase in the company's price, and/or an increase in the competitor's price can also be calculated.

B. *Assessing the combined effect of two or more variables:* A second class of information which can be extracted is the effect of the interaction of pairs of variables. For example, it is possible to calculate how much more effective is increased company advertising when the product price is \$5 as compared to \$10.

$$\begin{gathered} 1/4\,[\,(140\text{-}105) + (120\text{-}90) + (220\text{-}180) + (200\text{-}160)\,) \\ -\,1/4\,(\,(80\text{-}60) + (70\text{-}50) + (120\text{-}95) + (115\text{-}80)\,] \\ \times 1{,}000 = 11{,}250 \text{ units} \end{gathered}$$

In a similar manner other interactions, such as those between company and competitor prices or between company and competitor advertising, can also be calculated. In this example, the higher level of competitor advertising will reduce the effectiveness of company advertising by 1,250 units.

This approach has been developed into a general mathematical formulation by means of which much larger and more realistic problems can be attacked.

C. *Mathematical example:* Readers who remember something of college algebra will find the mathematics relatively easy to follow. Others can skip the following example without loss.

For the above problem, the generalized expression can be derived by multiplying:

$$(A_1 \pm A_2)\ (B_1 \pm B_2)\ (C_1 \pm C_2)\ (D_1 \pm D_2)$$

in which A_1 represents the first value and A_2 the second value of a variable, such as company advertising; and the other symbols represent the other variables. The effect of A_1, for example, is obtained by using a minus sign in the expression of $(A_1 - A_2)$ and plus signs in the others:

$$\begin{aligned} A = 1/4\,(&A_1B_1C_1D_1 + A_1B_1C_1D_2 + A_1B_1C_2D_2 + A_1B_1C_2D_2 \\ &+ A_1B_2C_1D_1 + A_1B_2C_1D_2 + A_1B_2C_2D_1 + A_1B_2C_2D_2) \\ &- 1/4\,(A_2B_1C_1D_1 + A_2B_1C_1D_2 + A_2B_1C_2D_1 + A_2B_2C_2D_2 \\ &+ A_2B_2C_1D_1 + A_2B_2C_1D_2 + A_2B_2C_2D_1 + A_2B_2C_2D_2) \end{aligned}$$

Substituting and letting A represent company advertising:

$$A = 1/4\,(105 + 180 + 60 + 95 + 90) = 11{,}250$$

which is the answer obtained above.

D. *Advantages of factorial analysis:* The technique of factorial analysis provides these benefits:

1. Maximum information for minimum work. In other words, it is the most efficient way of conducting tests.

2. A means of isolating the effects of each variable, even though several variables are acting simultaneously on the outcome. This is particularly important where some of the variables, such as weather, cannot be controlled.

3. A means for measuring the simultaneous interaction of any group of two, three, or more factors in affecting the outcome.

4. A means of eliminating bias from the individual observations.

5. A means of simultaneously testing several factors that may be suspected of influencing the process being studied.

As more variables are included in an experiment, the number of individual observations or tests required for a complete factorial analysis increases very rapidly. Under these circumstances it is possible, by good judgment, to reduce the total number of tests by eliminating many of the tests that would reveal only the more complex interactions of several of the variables and their effect on the outcome. Thus, it is often possible to reduce the size of the experiment without reducing the significant information derived from it.

E. *Potential applications:* Factorial analysis appears to be potentially a particularly useful tool in solving many marketing problems. This is because marketing decisions involve many interacting variables, most of which cannot be controlled and, therefore, are not amenable to simpler methods of analysis. The effectiveness of advertising is determined by a complex interaction of factors, some related to advertising and some to other marketing activities. The advertising factors include the media employed, the intensity of advertising effort, the copy used, and the advertising activities of competitors. The non-advertising factors include, among others, price, dealer discounts, marketing channels, sales promotions, services to dealers, style of product, and the many factors making up the market, such as age and income distribution, climate, and buying habits.

Similarly, decisions on pricing policy, sales promotion, style, and product diversification are all influenced by many variables which are not controllable but which interact in complex ways to govern the success of a given policy. Factorial analysis appears to provide an important tool for the solution of many problems of this type. Whether or not it can be used on a specific problem of this type will depend primarily on the availability of data.

Perhaps the most important conclusion at this point is that the proper design of a marketing test or study is essential if one is to obtain useful answers. The statistician must therefore be called in before the experiment is run, and must participate in the planning process. After the study is made, even the most expert statistician cannot squeeze meaningful conclusions from improperly selected data.

Analysis of the Data

In using these modern techniques, the statistician—and management—must never forget that statistical conclusions drawn from the analysis of data should always be checked against logic. If a high correlation has been observed in the past between two factors, there is a strong temptation to use this fact to predict the future. Yet, unless a logical reason for the observed relation can be found, it should be applied to predicting the future with extreme caution, if at all. For example, a high correlation is reported to have existed over the years between the number of Presbyterian ministers and the consumption of bonded bourbon. But the Presbyterian theological seminaries probably would be ill-advised to predict the number of applicants four years hence on the basis of newly distilled bourbon put into bond this year. Equally, the distillers probably can find more reliable measures of future sales than the annual admissions to Presbyterian theological schools.

An exceptionally good text is *Probability & Statistics for Business Decisions,* by Robert Schlaifer, McGraw-Hill, 1959.

Queuing or Waiting-line Theory

Queuing or waiting-line theory provides a means of predicting the probable length and delay of a "waiting line" formed by random arrivals at a servicing or processing facility of limited capacity. It is used in problems in which one or more elements occur at a rate that management can neither control nor even predict precisely.

Examples of Queuing Problems

A. *Assessing uncertainty in timing of ship arrivals:* Queuing theory is applicable to the kind of problem encountered at a crowded oil port, where port management can neither control nor predict precisely. Expected arrivals can, at best, be expressed statistically as an average rate of arrival for each week or month in the year. The average rate of arrival during a peak month might be known quite accurately to be 0.6 ships per day, based on the monthly oil delivery schedules and the capacities of the tankers being used. However, because of variations from schedule in actual departures from other ports, and because of variations in weather during the voyage, the time of each individual arrival is unknown.

If the ships were to arrive at a uniform rate, the minimum number of necessary loading positions, shore crews, pumping facilities, and storage tanks could be easily calculated. Given such uncertainty, however, the planning of loading facilities cannot be based on the assumption of a uniform rate of arrival; for when ships arrive randomly, there will be some periods in which arrivals are infrequent and servicing facilities will be idle, and other periods in which arrivals will be bunched and, as a result, some ships delayed in docking and loading. Clearly some excess servicing will be required if these delays are to be kept to a reasonable figure. For if, under conditions of random arrival, the service capacity is just equal to the dock capacity, the average number of ships which have to wait for dock space will tend, over the long run, to get longer and longer. Queuing theory provides the analytic tools for determining just how much excess servicing capacity is needed if over-all costs are to be minimized.

There are costs connected with the length of the waiting line and the time lost in waiting. There are also costs associated with increasing the capacity of the servicing unit, both capital costs and labor costs. Since arrivals are random, there may be times when there are waiting lines, and other times when there is idle servicing capacity. As the mean arrival rate approaches capacity, it can be known that, if that rate is maintained, the waiting line will tend to approach infinity as an ultimate limit. Clearly, then, capacity must be at least a little greater than the mean arrival rate. The optimum solution to this class of problems will provide a processing capacity just sufficiently in excess of the mean arrival rate to minimize the total of the cost of the added processing capacity plus the costs of waiting. It is also possible to determine the probable waiting time to be expected for each arrival for any given mean arrival rate and processing capacity.

B. *Manning a storeroom:* George Brigham, of Boeing Aircraft, has solved a problem of this type. Boeing has some 60 tool cribs which supply special tools required by mechanics. The problem was to determine the optimum number of clerks which should be on hand in each of the cribs to issue tools to mechanics. In a typical crib, it was found that, on the average, there would be 10.7 hours of work per 7.5-hour shift. Clearly this was more than one clerk could handle. If two clerks were used, there would be over four hours of idle time between them per shift.

It would seem that this idle time would be sufficient to make up for the bunching of mechanics resulting from a random pattern of arrival. But the application of queuing theory showed that this was not the case. If it is assumed that clerks are paid $2 per hour and mechanics are paid $5 per hour, the combined cost of the idle time of clerks and of the mechanics kept waiting would be $64 per shift for two clerks in the cribs, and $31 per shift for three clerks. For four clerks, the combined cost goes up again to $40. Thus it is more economical to have three clerks instead of two in the crib, even though they will, on the average, be busy only 47 per cent of the time. If the costs of mechanics relative to clerks were higher, or if the costs of delays in aircraft assembly resulting from delays of mechanics were included, the optimum number of clerks might be four or more, even though they would be busy only a small fraction of the time.*

C. *Other types of queuing problems:* There are many different types of problems for which queuing theory can provide the optimum balance between excess capacity and time lost in waiting. For example:

1. What is the optimum level of airport facilities to service aircraft arriving at unpredictable times?
2. What is the optimum capacity for repair shops in a major construction job?
3. What is the optimum size of a maintenance force which handles emergency repairs in a process industry?
4. Should a job machine shop expand to handle peak loads?
5. What stand-by equipment should be maintained against failure of production equipment?

Categories of Queuing Problems

There are many different types of waiting-line problems which can be solved by analytic means, although the difficulty of finding mathematical solutions increases as the complexity of the problem increases.

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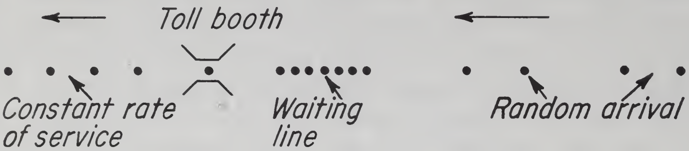
* George Brigham, "On a Congestion Problem in an Aircraft Factory," *Journal of the Operations Research Society of America*, Vol. 3, No. 4, pp. 412-428, November 1955.

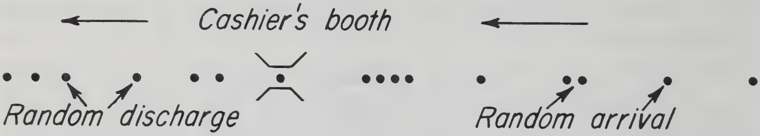
The basic types of queuing problems can be illustrated by a few simplified examples.

1. The arrival rate is random while the servicing time is uniform, as in the arrival of cars at a highway toll gate.



2. The average rate of arrival may vary with the time of day, the day of the week, or the season.

3. The servicing time as well as the arrival rate may be random, as is the case at a supermarket cashier's stand.



4. The service rate may vary with the length of the waiting line. For example, as the number of incoming orders awaiting processing in an office grows, the clerks will speed up their rate of processing.

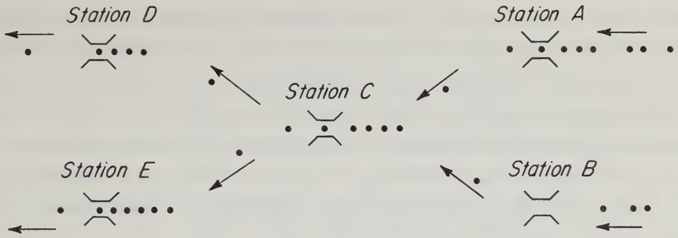
5. The arrival rate may vary with the length of the line. Most potential customers, on finding five cars waiting in line at a gas station, will drive on.

6. Priorities in servicing may be established. An aircraft arriving at a crowded airport with only 20 minutes' gas will be given landing priority over other aircraft which have been stacked longer.

7. The cost of waiting may vary with the length of waiting. For example, the cost of waiting in the delivery of perishable foods or newspapers may be at one rate for an initial period, and at a much higher rate as the food begins to spoil or a later edition comes out.

8. There may be more than one processing point, and shifts back and forth among waiting lines may be made as the relative lengths of the lines change.

9. Finally, more complex systems involving more than one class of items may be moved through a network of several processing stations, as in a machine shop, each with waiting lines varying both regularly and randomly.



In such a situation, it is possible to schedule service at each station in a variety of ways. Arrivals may be serviced on the basis of relative cost of waiting time, on the relative importance to an over-all activity, or on the probable length of servicing time required. Comparisons can be made of the efficiency of alternative systems of service scheduling.

Although one can never predict exactly the arrival of any single customer, queuing theory will make it possible to say, for example, that on the average a customer will have to wait 15 minutes, or that there is only one chance in ten that the customer will have to wait more than 45 minutes. Similarly one can conclude that half, or a third, of the customers will not have to wait at all.

Origin of the Theory

Queuing theory was originally developed in 1907 by A. K. Erlang of the Copenhagen Telephone Company as a means of estimating the amount of central switching equipment required. In the early thirties Eugene Grant of Stanford University again introduced the concept of queuing theory in economic equations that balanced the cost and capacity of a dam spillway with the cost and probable frequency with which floods will wash out the generator station below the spillway. But others did not have the imagination to see the general application of these concepts; and, as a result it was not until after World War II that further work was done on the theory, and further applications were made to practical problems in other fields.

Information Theory

Information theory provides a fundamental means of (a) measuring the information content of both symbolic and verbal languages and (b) relating the characteristics of an efficient communication system to the information content of the messages transmitted.

Developed primarily by Claude Shannon at the Bell Laboratories in the 1940s, information theory has made a considerable contribution to the more efficient design of electronic communication systems and electronic computers. There have been predictions that it would have equally important effects on organizational communications, and there have been several attempts to apply the theory to communication system involving human beings directly. So far, however, the results have been of limited significance.

Elements of Information Theory

A. *Capacity of communications link versus rate of transmission and interference:* Information theory states that the degree of error which can be expected in information transmitted through a communications link will be inversely proportional to the capacity of the link and directly proportional to the rate of transmission and to the interference or noise. For example, it is more difficult to understand a conversation in a noisy room, and doubly difficult if the speaker talks very fast, while it is relatively easier if he speaks more loudly.

B. *The binary code:* Information theory further has shown that the most efficient code for transmitting and storing information is the binary or two-digit code. The binary system and its decimal equivalents are shown below:

Binary	*Decimal*	*Binary*	*Decimal*
1	1	1001	9
10	2	1010	10
11	3	1011	11
100	4	1100	12
101	5	1101	13
110	6	1110	14
111	7	1111	15
1000	8	10000	16

C. *Redundancy:* It has been proved that the binary system is the most efficient of all possible systems for transmitting information. In comparison, all other systems, including verbal language such as English, have a high degree of redundancy and hence are relatively inefficient. But such redundancies have a saving characteristic in that they make it possible for the person receiving the message to correct many of the errors which may have occurred in transmission. For example, garbles in a telegram can nearly always be corrected by looking at the words or letters on either side of the mistake. Garbles in the transmission of numbers are more difficult to correct. If English were as efficient as a binary code, it would be impossible to know, by inspecting the message, that an error had occurred, much less to correct it.

This principle of redundancy has been applied to the design of self-checking codes, which will automatically indicate a large percentage of errors occurring in their transcription. This has been of considerable value in improving the over-all reliability of computers. A simple example will illustrate.

The number 3981 is to be transmitted. One additional digit is added so that this digit represents the sum of the other four. In this case 1 is added since the sum of the digits is 21, and the number with code becomes 1-3981. If any single one of the original digits is incorrectly recorded, the new sum will not end in "1" and an error will be indicated.

Limitations of Information Theory

The basic difficulty in using the concepts of information theory in improving the internal communication of organizations is that it provides no way of evaluating the relative importance of different types of information. Further, it is extremely difficult to convert many human communications to their symbolic equivalents. Because a computer deals only in numbers, and therefore all inputs must be so expressed, the binary code can be used internally in computers where it makes possible the storage of the same amount of information in a substantially smaller memory.

Conceptually, however, information theory can be of some value in studying human and organizational communication problems. For example, we can see that in general the greater the accuracy and the higher the speed required, the greater will be the costs. However, it is not possible, as it is for electronic systems, to use information theory to calculate the optimum balance between the costs of greater reliability and speed and the costs of errors.

Other efforts have been made to develop theoretical approaches to the optimum organization of communications in an organization. Jacob Marshak has developed a model of a simple decision problem in which the costs of obtaining additional information for a commodity-trading organization, as well as the costs of added internal communications, can be equated to the probable gains resulting from the information, and he is able to calculate for this very simple case that point beyond which the costs of additional information are no longer paid for by improved performance.

But until there are further major developments in theory which can handle the great complexity of management information systems, there may be few useful applications of information theories in the field of management.

Game Theory *

The theory of games of strategy, developed by the late John von Neumann and by Oskar Morgenstern, provides a rigorous means of determining, in certain situations of conflict, that strategy which is most likely to achieve the highest payoff. Owing to their formal rules of play and of payoff, games are by and large the simplest situations of conflict; hence it is logical that the first successes in developing a mathematically rigorous theory of strategy of conflict was in the field of games.

Because of present limitations on handling the complexity of actual conflicts, it is almost impossible to make practical applications of game theory to management problems. However, the executive who understands the concepts of game theory may find they provide helpful insights in solving some of his most difficult problems.

Finding the Best Strategy in a Conflict Situation

The following somewhat oversimplified example will demonstrate the basic elements of game theory. Two automobile-parts manufacturers supply practically all of the market for a given replacement part. Company A has always provided a design that could be used interchangeably on all General Motors cars, while Company B has always designed a part that could be used on all Ford and Chrysler cars.

By modifying his design, each part manufacturer can make his design usable on any of the other models. However, in so doing he will make his product somewhat less than ideal for his primary customer. Thus, if Company A designs a product that also can be used on Plymouths, it will lose some of its advantage in the General

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* Game theory is quite distinct from operational gaming (a trial-and-error method of simulating problems in which human players are involved), and the two should not be confused.

Motors field, should Company B decide to invade the General Motors field. Each year the two companies analyze the designs for new models and decide upon the design they will use in the new year. Neither company can know the other's plans early enough to modify its own decisions. Hence neither can select its own strategy to take advantage of the other's decision.

In this example, it is assumed that the unit profit remains constant and that the objective of each company is to maximize its total sales. For simplicity, each company is assumed to have only three alternative designs.

For each pair of decisions by the two companies, there will be a different net result or payoff. In some cases, the combination will favor one company and in some cases it will favor the other. Sometimes there will be a single preferred strategy for each company; in other situations a mixed strategy will be preferable.

A. *When there is a single preferred strategy:* In this example, if Company A chooses Design a_1 and Company B selects Design b_1, it is estimated that Company A will gain 100,000 sales while Company B will lose a net of 100,000 sales. If Company A chooses Design a_2 and Company B again selects Design b_1, Company A will now suffer a net loss of 20,000 sales to Company B. The payoff for each of the other pairs of decisions of A and B is shown below.

		Company B				
		b_1	b_2	b_3		
	a_1	10	2	4	2	
Company A	a_2	−2*	0	−8	−8	(add 0000)
	a_3	−5	1	8	−5	
		10	2	8		

Analysis of this payoff table, or matrix, will show that in this case there is a *single preferred* strategy for each company. Company A can always be sure of gaining at least 20,000 sales if Design a_1 is chosen. If instead, Design a_2 is selected, Company A may lose as many as 80,000 sales if Company B decides on Design b_3. If design a_3 is chosen, Company A may lose as many as 50,000 sales.

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* Negative numbers indicate loss of sales by Company A to Company B.

Similarly, Design b_2 will minimize Company B's losses at 20,000. Selection of Design b_1 could result in a loss of 100,000 sales to Company A, and Design b_3 could result in a loss of 80,000 units. Thus the preferred strategies for both companies will result in an expected gain for Company A of 20,000 sales. The pair of decisions, a_1b_2, represents a "saddle point," which is a point where the largest of the minimum values in all the rows is equal to the smallest of the maximum values in all the columns.

B. *When a mixed strategy is preferable:* If the matrix looks like the one below instead, there is no longer a single preferred strategy for both companies.

		Company B		
		b_1	b_2	b_3
	a_1	2	−4	2
Company A	a_2	4	−6	−4
	a_3	0	2	1

Pay-off matrix for Company A

Examination of the foregoing matrix will show that there is no single combination of decisions which will always be advantageous to each. Under these circumstances, Company A can assure at least a minimum figure by adopting a mixed strategy. That is, Company A sometimes picks one alternative, and other times another. Company B will be deprived of the opportunity of capitalizing on A's strategy. For example, if B knows from past experience which decision A will make, B can always win. Therefore the only hope of A is to mix his strategies so that B remains in the dark until it is too late for B to change his own plans.

C. *Choosing the best strategy mixture:* It can be shown that, for each game in which there is no one preferred strategy, the strategies should be mixed in specific proportions which will be determined by the numerical values in the payoff matrix. In order to remove any pattern in the sequence of strategies selected, each decision on a given play should be made by random selection. For example a single card might be selected from a deck in which the number of cards representing each alternative was in proportion to the ratios of the mixed strategy.

In this example, the proportions or odds for each strategy available to A are shown below.

		Company B			
		b_1	b_2	b_3	*Odds for A*
	a_1	2	−4	2	1
Company A	a_2	0	2	1	1
	a_3	4	−6	−4	8
		19	7	4	

If Company A makes its decision by random choice weighted in the ratio of 1:1:8, and Company B adopts a pure strategy of picking alternative b_1 all the time, Company A can expect to gain, on the average, 600 in sales at the expense of B. Similarly, against any other single strategy or any mixed strategy of B, Company A can expect the same gain.

If, however, Company A were to abandon its optimum strategy and instead adopt a strategy of weighting each strategy equally, while Company B employed its optimum mixed strategy, Company A could expect an average decrease in sales of 6,500. This could be worse than settling on Design a_3 permanently, where one could at least be sure of losing no sales regardless of Company B's strategy. Similarly, every other strategy which Company A might select will bring a probable return of less than the optimum mixed strategy described above.

Practical Limitations of Game Theory

Game theory has not progressed very far in its ability to handle the complex problems of actual competition. The example used above belongs to the simplest type, in which there are only two conflicting interests and in which the losses of one are always exactly equal to the gains of the other. This is termed " a two-person zero-sum game." Nearly every conceivable practical application of game theory to management decision making involves a non-zero-sum solution because the gains of one are seldom equal to the losses of the other. Often the number of interests involved will be three or more, and the number of strategies available to each will be large. All of these factors complicate greatly the solution—both in theory and in computation. Even the use of computers does not significantly reduce the difficulties.

Insights Provided by Game Theory

Because of the difficulties in handling complex problems, it seems unlikely that game theory will produce complete answers to very many practical problems. Game theory does, however, provide important insights into situations of conflict, insights which will be of considerable value in approximating optimum solutions by means less rigorous than formal game theory. John Williams, head of the Mathematics Department of the RAND Corporation, writes:

> While there are specific applications today, despite the current limitations of the theory, perhaps its greatest contribution so far has been an intangible one: the general orientation given to people who are faced with overcomplex problems. Even though these problems are not strictly solvable—certainly at the moment and probably for the indefinite future—it helps to have a framework in which to work on them. The concept of a strategy, the distinctions among players, the role of chance events, the notion of matrix representations of the payoffs, the concepts of pure and mixed strategies, and so on give valuable orientation to persons who must think about complicated conflict situations.*

In summary, game theory provides the following insights into situations of conflict:

1. Under certain conditions there will be a single best strategy for each interest involved. The best strategy will either assure a maximum profit or will minimize the damage an intelligent opponent can do, regardless of the strategy he employs.

2. Under circumstances in which any single strategy will leave one vulnerable to the worst the opponent can do, a mixed strategy is the best course of action. For each specific situation, there will exist an optimum mix of alternative tactics which will maximize one's probable gains (or minimize one's possible losses) regardless of the tactics used by the opponent.

3. Game theory provides a conservative strategy which is primarily of value against a skillful opponent. Against a careless opponent or when information about the opponent's probable strategy can be obtained, it should be possible to exploit such weakness by a more aggressive strategy than game theory provides. A good poker player employs a mixed strategy of bluffing and not bluffing, but he also makes use of his psychological analysis of his opponents' weaknesses.

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* J. D. Williams, *The Compleat Strategyst*, McGraw-Hill, 1954.

MATHEMATICAL SOLUTIONS TO PROBLEMS OF GREAT COMPLEXITY

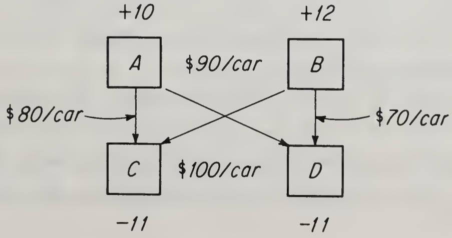
Linear Programing

Linear programing is an analytical method of finding, from a large number of feasible combinations, the optimum combination of several limited resources to achieve a given objective, such as maximizing profit. A linear programing problem is described by algebraic equations in which the symbols represent quantities of such things as raw materials and machine capacity and solved by the algebraic manipulation of these equations. The answer is the optimum combination as measured by an algebraic statement of the payoff criterion.

Linear programing is one of the few really new tools of mathematical analysis for decision making which have been developed in the last ten years. In nearly every other application of mathematics to decision making, the basic mathematics have been in existence for many years. Yet linear programing has had, next to the mathematics of probability, more practical applications to management problems than has any other single mathematical technique.

The great advantage of linear programing is that it is a sophisticated and very efficient shortcut in mathematical calculation, which makes it possible to solve in a few hours a problem that, if conventional means of calculation were used to find the best solution, would require an impossibly long time.

Some problems of the type that linear programing can solve are so simple that no special technique is needed. If, for example, there are surpluses of freight cars at two cities that are required to meet shortages in two other cities, one can easily determine how to redistribute these cars so as to minimize the costs of hauling empty cars. Assume that City A has a surplus of 10 cars and City B a surplus of 12 cars. City C and City D require 11 cars each. It costs $80 to send a car from A to C, $100 from A to D, $90 from B to C, and $70 from B to D. The problem can be pictured as:



In this problem, it is obvious that it will be least expensive to send all 10 cars at A to C and that one additional car is needed to be sent from B to C, while the balance of 11 at B should be sent to D. (Even on this simple problem, however, if one were to calculate all feasible solutions and to find the cost of each, 11 different shipping plans would have to be examined.)

But if, instead, one had to solve a similar problem involving three origins and five destinations, with a total of 20 cars to redistribute, the answer no longer would be apparent by inspection. If one were to list and evaluate all feasible solutions to this relatively simple problem, the number of solutions to be considered would be in the order of 10,000,000. Linear programing can provide the solution in not more than six or seven steps.

If the problem involved 50 to 100 origins and destinations and several thousand freight cars, the number of possible solutions would be beyond counting, and the linear programing solution would only be possible if one used a high-speed electronic computer. Fortunately, standard solutions to linear programing problems have been programed for most of the computers in use, and with such a program the computational time is cut to a very short period.

Types of Linear Programing Problems

The three general types of linear programing problems are described below.

A. *Transportation and assignment problems:* In essence, transportation problems (like the one described above) involve the movement of limited quantities of a single product from a group of origins to a group of destinations. The objective may be stated in a variety of ways: to minimize the total distance traveled, to maximize total profit, to minimize the total time involved, or to minimize the total costs.

The assignment problem is a variation of this class of linear programing problems. Here one is confronted with the assignment or allocation of one class of resources or products among another class of uses so that the over-all program is optimized. Two examples will illustrate assignment problems:

A sales manager is faced with the problem of *assigning salesmen to territories.* Each salesman has particular characteristics that make him better suited to some territories than to others. But, as some salesmen are also generally superior to others, the sales manager cannot have the single best man in each territory. His problem is to assign the best salesman to a particular territory and to assign the second or third best to the remaining territories so that total sales will be maximized. If—and his is an important "if"—the sales manager is able to set down some quantitative measures of the relative productivity of each salesman in each territory, he can use a simple technique of linear programing to find the optimum assignment pattern. This requires that he be able to make reasonably accurate estimates of the return on sales that can be expected from each salesman in each different territory.

Similarly, a large contractor is faced with the problem of *assigning his fleet of trucks* of different types to a number of different construction jobs. Each type of truck has its particular features with regard to capacity, operating costs, and loading and unloading time. Using linear programing techniques, a solution can be found for the assignment of these trucks which will minimize the total cost of moving earth.

The potential value of this technique is demonstrated by the fact that a simple problem involving the assignment of only 12 units to 12 jobs would otherwise require the examination of about 479,000,000 different combinations of assignments or programs, if all possibilities were looked at. This comparison is possibly misleading in that most of 479,000,000 combinations could be eliminated as obviously inefficient. But the number remaining would still be very large.

The assignment of different types of manufacturing equipment to different jobs may be approached in the same way. The principal limitation on this solution is the necessary assumption that all assignments are made for the same time period. It cannot handle the complex type of assignment problem in which one machine may alternately be performing several different types of operations within a single time period, or in which all of the jobs are not started at the same time.

B. *Problems of allocating limited resources among competing activities or uses:* In this class of problems, inputs are specified as being "not more than" specified total quantities of each input available to be divided among the competing uses. The optimum solution may thus require the full amount available or may require a stated lesser quantity.

It is interesting that the first formulation of a linear programing problem of this type was made by George Stigler in 1945. Stigler's objective was to find that diet which would provide, for a minimum total cost, specified minimum quantities of key vitamins, minerals, and proteins, as well as a minimum caloric intake. Each individual food considered provided a different combination of requirements and at a different cost. Paying no attention to taste or variety, he found that, at 1944 prices, his minimum diet requirements could be met for $59.88 per year by a diet of cabbage, beans, and wheat.

The blending of gasolines was one of the earliest industrial applications of linear programing. It was quite similar to the diet problem in that it involved the blending of several intermediate refinery products, each with different quality characteristics, costs, and availabilities, to produce a range of final products. In this problem, each of the inputs was available in quantities up to specified amounts and each of the products could be sold at stated prices up to specified maximum quantities. In one actual solution, it was found that the maximum profit would be realized if less aviation fuel were produced than could be sold and if part of the lower-grade inputs were diverted to motor gasoline—even though it was technologically feasible to meet total aviation gasoline requirements.

Mathematical example:

Assume that the owner-manager of a small specialty machine shop has demand for two products, A. and B. The two products each require different amounts of time in the stamping, milling, drilling, and paint departments, as shown in the following table. The table also indicates the total amount of time available per week in each of the departments. Product A yields a contribution to profit and overhead of $5/unit and Product B yields $4/unit.

CAPACITY UTILIZATION TABLE

Process → Part ↓	Stamp	Mill	Drill	Paint	
Product A	30	60	40	12	Usage of each process by products A & B expressed in minutes/part
Product B	40	20	25	10	
Total process capacity	12000	12000	10000	6000	Total minutes per week available in each process

The problem is to determine the mix of A and B such that total contribution to profit and overhead will be maximized, without exceeding the available capacity in any of the machine shop departments. (Maximum possible sales per week of each product could also be restrictions in the problem, but have been omitted to simplify the illustration. It is also assumed for simplicity that all costs other than shop processing are identical for Products A and B, and hence need not be considered in the profit maximization problem.)

The first step is to express the problem in the form of linear equations. Let X_A and X_B equal the quantities of A and B to be produced. Since A and B yield \$5 and \$4 per unit, respectively, our objective is to maximize the expression:

$$5X_A + 4X_B,$$

subject to restrictions which assure that we do not choose a solution which will put demands on any of the process departments in excess of their available capacity. Referring to the capacity utilization table, it can be seen that these restrictions can be written as inequalities (since it is feasible to choose a solution which uses less than the available capacity), as follows:

Stamping $30\ X_A + 40\ X_B \leqq 12000$

Milling $60\ X_A + 20\ X_B \leqq 12000$

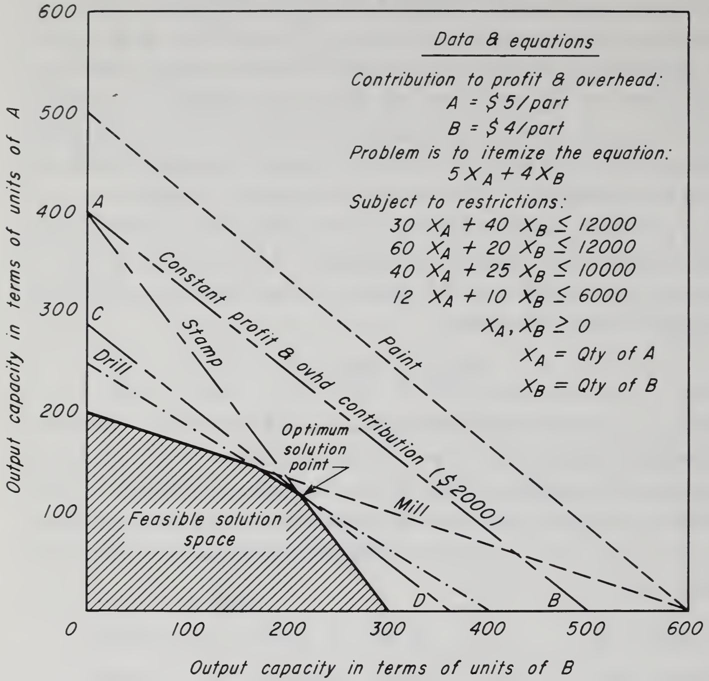
Drilling $40\ X_A + 25\ X_B \leqq 10000$

Painting $12\ X_A + 10\ X_B \leqq 6000$

Furthermore, since Products A and B require processing in all departments during the course of manufacture, *all* of the restriction conditions expressed above must be satisfied for any mixture of A and B chosen as a possible solution. The problem is to find the mixture which yields the maximum profit.

Since this example involves a mixture of only two products, we can illustrate the solution graphically. The first step in this process is to "map" the restriction equations in terms of quantities of A and B. (refer to Figure 1.) For example, the paint shop has a capacity of 6000 hours per week. Thus, if painting were all we were concerned with, we could produce 500 A's ($6000 \div 12 = 500$) or 600 B's ($6000 \div 10 = 600$), or any mixture of A's and B's which would not use more than 6000 minutes per week of painting time. Thus any point on the line labeled "PAINT" on the graph, or any point under

Fig. 1 Linear programming example
optimum mixture of products A & B



the line, is a "feasible" solution with respect to the restriction on painting. Similar lines must be drawn for each of the other restrictions. Since all restrictions must be satisfied simultaneously, any feasible solution to the problem must lie under *all* of the restriction lines, and hence must be in the area or on the boundary of the area labeled "FEASIBLE SOLUTION SPACE."

Having defined the area containing all feasible solutions, the next step is to locate the point or points which are *optimum* in terms of the objective desired. This is done as follows:

1. Draw a "constant profit contribution" line. For example, we have chosen arbitrarily to draw a $2000 line. Thus any point along the line A-B on the graph would represent a mixture of A's and B's which would provide $2000 contribution to profit and overhead. The end points of this line are determined as follows:

$$\text{If } X_B = 0\text{, then } X_A = \frac{\$2000}{\$5} = 400$$

$$\text{If } X_A = 0\text{, then } X_B = \frac{\$2000}{\$4} = 500$$

The purpose of drawing the "constant profit" line is to determine its *slope,* since any "constant profit" line will be parallel to the one we've already drawn as long as the realtionship between the profit contributions of A and B remain constant; i.e., 5 to 4.

2. Since the line we have drawn does not touch the "FEASIBLE SOLUTION SPACE," the level of profit arbitrarily chosen was too high. Therefore, the next step is to "move" the line down, remaining parallel to the original \$2000 line until some part of the line first touches the FEASIBLE SOLUTION SPACE. The line C-D represents this condition, and hence the point where $X_A = 120$ and $X_B = 210$ is the optimum solution to the problem.

Total contributions to profit and overhead at the optimum solution may be determined by solving the original objective expression. Thus:

$$\begin{aligned} 5X_A + 4X_B &= \$5(120) + \$4(210) \\ &= \$600 + \$840 = \$1440/\text{week.} \end{aligned}$$

It is a characteristic of linear programming problems that the optimum solution will always be somewhere on the boundary of the feasible solution space.

A simple graphic solution was chosen here to illustrate the concept of linear programming. In general the determination of optimum solutions in problems of this type is extremely complex, since graphic solution is impossible if the "product mix" exceeds two or at most three items. Some very practical problems involve literally hundreds of "products." Solution of these complex problems is obtained through use of the so-called *simplex* technique (although it is not a "simple" technique). It is an iterative process by which solutions closer and closer to optimum are obtained at each step until the provably best solution is reached. The process is a purely mechanical one for which precise step-by-step rules exist. The technique has been programmed for almost every general purpose digital computer in existence, and the routines are readily available from computer manufacturers or program libraries.

The numerical solution of the problem is achieved by the so-called *simplex* technique, which, as stated above, is an iterative process to obtain solutions closer and closer to optimum until the provably best solution is reached. The process is a purely mechanical one for which precise step-by-step rules exist. In this respect the process is similar to the process of finding square roots by hand.

2. *Examples of applications:* A typical linear programing problem involving the operation of a chemical plant might be formulated as follows:

(a) Ten different basic chemicals are used in the production of eight separate products.
(b) Each of the ten raw materials can enter into each of the final products. The relative proportions of raw materials used in each product can be varied within stated limits. In other words, within limits, each of the raw materials can be substituted for one or more of the others.
(c) Maximum capacities exist for each production unit, and each production unit can produce alternative products at known costs.
(d) Each of the raw materials is available up to specified amounts at given base prices; additional quantities are available at premium prices.
(e) A maximum market exists for each of the products at a given price; additional quantities can be sold only at discounts.

Given these conditions, it is possible to find the most profitable production plan including:

(a) The amounts of each raw material required
(b) The proportions of each raw material used in each product
(c) The amount of each product produced
(d) The profit which will be realized

In addition to finding the optimum production pattern, it is possible to determine quickly the profit implications of:

(a) Changes in prices or available quantities of raw materials
(b) Changes in demand for products in terms of price and quantity
(c) Changes in production technology
(d) Additions to production capacity

Another type of application is in *the allocation of limited capital among various items of inventory.* In this situation, inventories cannot be maintained at ideal levels because of the overriding limitation on capital. The optimum allocation will minimize the total expected losses resulting from inadequate inventories and runouts. Some losses may be accurately predictable, and others will be subject to random withdrawal of items from inventory. Predictable losses include those arising from uneconomical shipments. Losses arising from random withdrawal from inventory, such as those resulting from sales or from failure of a part, often can be described by probability distributions. If such random elements are present, the computation of the solution becomes more difficult and the answer is expressed as "the most probably optimum."

C. *Problems involving time*: In scheduling production to meet a sales forecast, random irregularities and seasonal variations in sales can be met by any combination of changes in the level of regular production, in the amount of overtime production, and in the level of inventories. For any given number of weeks or months ahead, it is possible to determine that production schedule which will meet the sales forecast at the minimum total cost of production and warehousing.

D. *Other types of problems:* Linear programing has also had limited application to problems in which some of the factors can be described only as a *range of probabilities* for a corresponding range of values. For example, a distribution problem may involve demands on each of several warehouses which are subject to random fluctuations and that can be expressed only as probabilities. Solutions to this type of problem that do not substantially increase the size of the computational effort have been worked out by George Danzig of the RAND Corporation.*

In another application, linear programing has been used to recognize the *production, warehousing, and distribution* of a large paper company which produces the same types of paper in several different plants and on different types of machines. By means of linear programing, a production and distribution program for each mill was worked out so that the over-all performance of the system was optimized. This involved several major variables:

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* George Danzig; "Recent Advances in Linear Programing," *Management Science*, Vol. 2, No. 2. January 1956.

(a) Each plant had different models of paper machines, each of which had different operating costs and capacities for each grade of paper.

(b) Requirements of each distribution warehouse varied both as to type and volume of paper sold.

(c) The paper mills and warehouses were widely scattered throughout the United States and Canada, and transportation was an important expense.

The final answer provided the optimum schedule for each of the plants, and showed, for each grade of paper, whether production should be concentrated in one plant or in more than one plant.

Still other types of problems for which linear programing has been used or appears feasible, include:

(a) Machine loading. (under rather restricted conditions).

(b) Blending of commercially produced livestock feeds to provide required nutrient value at minimum cost.

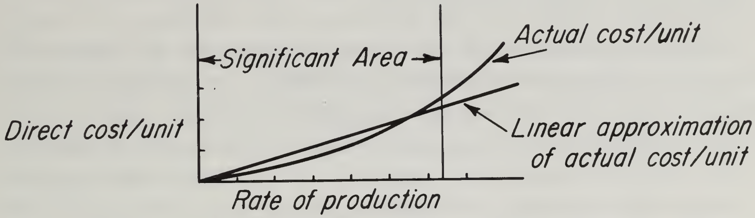
(c) Scheduling of air cargo operations involving several different types of aircraft, cargoes, origins, and designations.

(d) Analysis of competing bids where bidders are permitted both individual group bids and where individual bidders' capacities are limited so that only certain combinations of bids from a single supplier can be accepted. Linear programing makes it possible to award contracts so that overall cost is minimized and so that no contract exceeds the capacity of the individual supplier.

(e) Maximization of over-all return by the optimum distribution of product among the major population centers. For example, a national distributor of fresh fruits has a crop that varies considerably, both seasonally and from year to year. If he can successfully approximate the price-volume relationship for each distributing center, he can use linear programing to tell him how to divide any given total volume so that his price in each center is roughly in line with other prices (after subtracting freight) and so that his total profit is maximized.

Limitations of Linear Programing

Linear programing as a tool has limitations as well as advantages. The principal limitation is that it treats all relationships as linear. For example, if 100 square feet of storage space costs $150, then 200 square feet costs $300, and 30 square feet costs $450. If the direct costs on 10 units are $100, the direct costs on 20 units are assumed to be $200. In actuality, the unit costs of storage may go up as space becomes scarcer; the direct costs of production may increase as output approaches capacity; or the cost of a lost sale may be greater for the 50th unit than for the first. This diffiulty is met in part by selecting the straight line which, in the critical range, most closely approximates the true curve.



A second limitation on linear programing is that it is very difficult to handle more than one set of conditions at a time. In other words, it can provide the optimum allocation of resources for one set of conditions, but it cannot, without extreme difficulty, find the optimum program over several sets of different conditions. For example, if the oil refinery problem, mentioned above, extended to cover 12 monthly periods in which there were different prices, availabilities, and demands in each period, it would be very difficult to use linear programing.

A third difficulty is common in all applications of mathematical analysis to management problems. Most management problems of real importance quickly grow to such size, as measured by the number of factors or variables involved, that even the largest computers cannot handle them. This means that simplification of the problem, by cutting out some of the less important elements, becomes essential. Here the skill of the trained mathematical analyst becomes all-important. He must be able to cut the problem down to size without destroying its essential characteristics, or else to conclude that the necessary simplifications would destroy the significance of the conclusion.

Finally, like most analytical tools, linear programing usually entails a costly data-collecting program. It requires, to set up the problem, a skilled mathematical analyst, who can determine what simplifications can be made safely in order to make the problem manageable and who can program the solution so that the maximum of useful information can be provided at the minimum cost. In some cases it has been found that older cut and try methods of scheduling production, especially where done by the same person over a period of years, often have been coming quite close to the optimum as subsequently determined by a linear program. Linear programing, therefore, should not be undertaken without a competent man to direct the program and unless the process to be studied has the possibility of saving substantial sums of money.

Advantages of Linear Programing

Linear programing also has several advantages, in addition to its primary capability of precisely handling a very complex problem involving a very large number of separate variables. Once a problem has been set up and solved, it then can be modified with very little additional work. For example, if the cost of a raw material changes, it is quite easy to substitute the new price and find the new optimum without re-solving the entire problem.

Linear programing also provides a new approach to the problem of cost analysis. If linear programing has been used to find the optimum program for the manufacture of a given mix of products going through a plant, it is relatively simple to determine the additional costs involved in adding one more order to the mix of orders in process. This is of considerable value to a firm which may be bidding on a new job and wishes to know the real added costs involved for a given mix or orders already in process. For certain types of operations, linear programing thus provides an alternative approach to the sticky problem of joint costs and allocation of indirect expense.

Similarly, linear programing can be used to determine, for any specific mix of products, the added, or decreased, income resulting from: (a) The addition of machines, (b) the replacement of one machine with another, (c) a change in price of one or more raw materials, (d) a change in market demand or price.

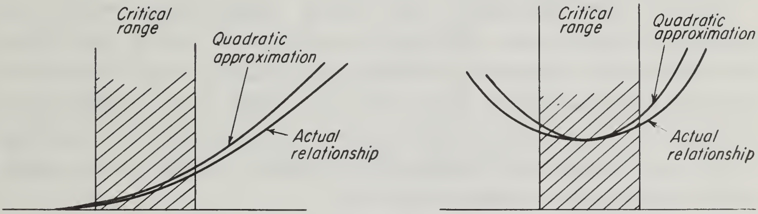
A linear program can also be used to find the actual costs of producing items for good will or maintaining a complete line to offer customers. It can also show more accurately than cost accounting the most profitable lines on which to concentrate sales effort and marketing expenditures.

In summary, once set up, linear programing may be an important tool to a manufacturer to permit the quick assessment of the profit implications of any sudden changes in the market situation and to make the best decisions on how to adjust his operations to meet these conditions.

Quadratic Programing

Like linear programing, quadratic programing is an algebraic technique. A quadratic equation differs from a linear one in that it includes a second order, or "square" term; for example, $X^2 + 4ax + b = c$.) The difference is that quadratic programing can handle problems in which the cost relationships can be nonlinear. For example, direct costs per unit can change with volume. Thus quadratic programing can be used to find the optimum schedule of production or distribution activities when direct costs per unit are not constant, but change with volume. It is also of value in finding the optimum schedule over successive time periods when demand fluctuates.

Fig. 4



The advantage of the quadratic approximation of the actual cost relationship as compared to a linear approximation is shown in Figure 4. The specific proportions of a quadratic curve may be changed to fit any particular cost relationship. If the cost relationship is actually a straight line, the quadratic curve may be flattened out so that it closely approximates a straight line in the critical range. On the other hand, if the actual relationship is sharply curved, the quadratic curve can be closely fitted to it.

Charles Holt, Franco Modigliani, and Herbert Simon of Carnegie Institute of Technology have developed in quadratic programing a form of analysis which: (1) by use of nonlinear (quadratic) curves, permits a closer approximation than does linear programing of the actual cost relationships, and (2) allows the computational work to be reduced to a linear form and hence made relatively simple.[*]

[*] C. C. Holt, F. Modigliani, H. A. Simon, "A Linear Decision Rule for Production and Employment Scheduling," *Management Science*, Vol. 2, No. 1, Oct. 1955, p. 1. Also, C. C. Holt, F. Modigliani, J. F. Muty, "Derivation of a Linear Decision Rule for Production and Employment," *Management Science*, Vol. 2, No. 2, January 1956, p. 159.

Adjusting Production and Inventory Levels to Changes in Sales

One successful application of this technique to production scheduling has provided a means for setting over-all production and employment levels which take into consideration both forecasts of changes in future demand and current levels of inventory, work force, and production. Where demand for a product fluctuates markedly, three alternative ways of absorbing these fluctuations are open to management:

1. Adjusting the size of the work force by hiring and firing in exact conformity with the fluctuations in orders
2. Adjusting the production rate to fluctuations in orders by working overtime or "undertime" with a constant work force
3. Allowing inventories and the backlog of orders to fluctuate while maintaining a constant work force and production rate

Each of these alternatives has certain disadvantages, which are reflected in added costs. Adjusting production by hiring and firing is expensive in personnel processing and training—and in morale. Overtime and undertime both result in added costs. Leveling production by an inventory cushion adds to requirements for working capital and warehouse facilities. Back-ordering results in lost sales and in disrupted production schedules.

By means of quadratic programing, it is possible to determine how much of the fluctuation in forecast demand should be met by changes in the work force, by changes in overtime, by changes in production rates, and by changes in inventory levels. The objective is to smooth these changes in a way that minimizes total costs over the long run.

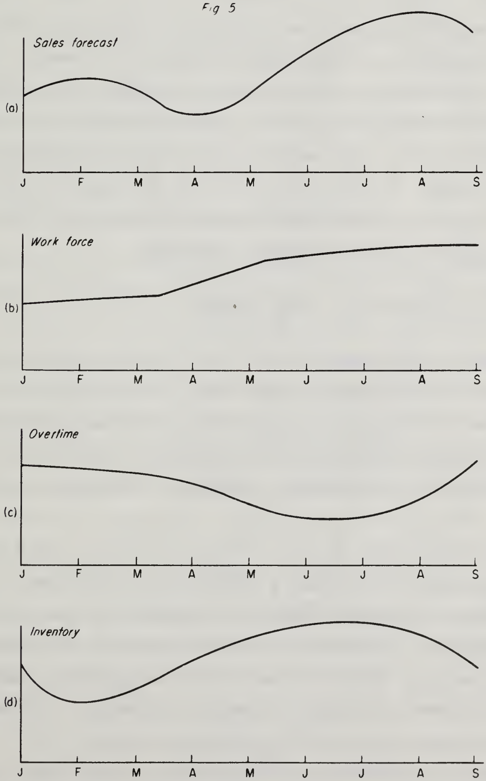
It is thus possible—at the beginning of each week, month, or quarter—to adjust the combination of size of work force, overtime, and production level so that (1) immediate requirements are fulfilled; and (2) current levels of production place the company in the best position to meet forecast demands for succeeding periods at a minimum cost.

Adjusting Production to a Sales Forecast

As shown in the example in Figure 5a, the sales forecast of a chemical manufacturer shows a small rise in demand in February, a drop through the spring, and a substantial rise through the summer into the fall. The application of quadratic programing might respond to this forecast by maintaining the same work force until

March (Figure 5b); adding labor during April and May in anticipation of the rise in demand in the summer and fall; and stabilizing the payroll from May on. Overtime (Figure 5c) would not be increased to respond to the temporary February rise in demand; instead it would be cut, because the mathematical program anticipates the subsequent rise in demand in the summer by calling for more regular workers to be hired in March, thus tending to decrease the need for overtime. The February rise in demand would thus be met primarily from inventory (Figure 5d) while the July-September increase would be met primarily by increasing the size of the labor force beginning in March. Only in August would some increase in overtime be called for so that the permanent work force could be stabilized at a level below the August peak.

Fig 5



A Test of Quadratic Programing

In the application of quadratic programing to the scheduling of a paint company, a minimum saving of 8:5 per cent of total direct costs was shown to be possible, even when using a primitive forecasting technique. In this case, quadratic programing was used to "reschedule" past production so that the costs could be compared with the costs of the schedule actually used. In this test, the "forecast" of future orders was, of course, not the actual demand that subsequently developed. Instead it was simply a running average of the previous three months' demand. In actual practice it was impossible to use considerably more accurate forecasts, so it is reasonable to believe that actual savings would be above the 8.5 per cent figure. The technique is now being used in the regular scheduling of the plant.

In checking the performance of quadratic programing in scheduling the operations of the paint factory, it turned out that if an absolutely perfect forecast had been available, the added improvement in performance would have been less than the improvement already obtained by the use of quadratic programing with an indifferent forecast. In this case at least, trying to perfect the forecasts is of less importance than improving the means by which production is adjusted to imperfect forecasts.

Advantages of Quadratic Programing

One unique advantage of the quadratic programing technique is that forecasts of future orders may be expressed by probability distributions as well as by single values, without adding significantly to the computational work. This capability enhances the usefulness of the technique in providing answers to problems involving uncertainty.

A further advantage of this technique is that a quadratic program, once set up, can be re-solved periodically to take into consideration new and better forecasts. Thus it is possible to correct the schedule as time progresses. But each new solution will, of course, take into consideration past decisions so that changes in production levels and in labor force are kept to optimal values by avoiding swings in inventories, labor force, and production rates.

The potential applications of quadratic programing appear to be broader than the examples cited. Among other possible uses are:

financial programing, including cash flow, short-term borrowing, and long-term financing; the programing of equipment expansion and replacement; and the scheduling of ships and other types of transportation. Industries that have guaranteed annual wage plans and that also face fluctuating sales may find quadratic programing of particular value in stabilizing production and employment levels.

Dynamic Programing

Dynamic programing is a method of solving multistage programing problems in which the decisions at one stage become the conditions governing the succeeding stages. (Quadratic programing is a form of dynamic programing in that it can be used to find that set of decisions over successive periods which will minimize total costs.) If, for example, there were no added costs or other disadvantages associated with changing production and employment levels from week to week, a factory could be scheduled on a weekly basis with no regard for the level of production during the preceding or following week. It is the interdependence of decisions at several stages that distinguishes dynamic programing problems.

Scheduling and Locating a Series of New Plants

In the planning of a capital expansion program, several alternatives are usually available to management. If a single new plant is built in the center of a large market area, the size and location of subsequent plants in the area will be different from the size and location of plants if the first plant had not been built. If two smaller plants are built instead of one larger one, the possibilities for further construction are quite different. Very quickly the number of alternative combinations of plants and their construction schedules becomes very large, and it is extremely difficult to evaluate all alternative combinations.

In order to evaluate all feasible combinations of schedules and locations, a very substantial, if not impossible, computational job is required. Moreover, the problem is complicated by the fact that, since any given pattern of plants will only be optimal at a given point in time, any expansion plan will result in less than optimum operations at any given time, simply because plants cannot be relocated every year. Dynamic programing provides a possible means of determining the sizes, locations, and timing for a series of plants which, taken together, will optimize the over-all profits of the enterprise.

Potential Uses of Dynamic Programing

Dynamic programing is therefore of considerable potential value in a large class of critical management problems. But whether or not a given problem can be solved today, even by approximative techniques, can only be determined by detailed examination of that specific problem. Some of the types of problems which may be solvable include:

1. Long-range capital budgeting
2. Timing of equipment replacement
3. Machine loading in job shops
4. Transportation scheduling to meet shifting demands
5. Smoothing of production levels to meet variable demands
6. Allocation of limited resources between current consumption and reinvestment to increase future output

Since dynamic programing is clearly in the developmental stage, its applications should be limited to situations of high potential payoff and situations where management is prepared to regard the necessary expense in the same light as it would research-and-development expenses. Because of the complexity of most managemnt problms (discussed later) will provide a more practical approach to most problems than will formal dynamic programing. The essential difference is that dynamic programing attempts to provide the exact optimum by the solution of a set of mathematical statements or equations. Simulation, instead, is a controlled trial-and-error process in which successively better numerical answers are obtained by repeated trial solutions. The process is repeated until an answer is found which approximates the true solution sufficiently closely to be acceptable.

Input-output Analysis

Input-output analysis is an analytical method of determining the change in the direct and indirect requirements for goods and services which will result from a change in demand for other related products. The mathematical solution of this type of problem is accomplished by use of matrix algebra—a special branch of mathematics—which describes the problem as a matrix in which the input-output relationships among each pair of products, such as coal and steel, are entered in the appropriate cells of the matrix.

Analysis of an Entire Economy

Input-output analysis was originally developed by Wassily Leontief of Harvard University as a tool for analyzing an over-all economy. Mathematically, it is a variation of linear programing and provides a quantitative framework for the description of an entire economy. Basic to input-output analysis is a unique set of input-output ratios for each production and distribution process. For example, the inputs of coal, ore, limestone, electric power, etc., all enter into the production of pig iron in fixed ratios. Thus, if the ratios of inputs per unit of output are known for all production processes, and if the total production of each end product of the economy—or of that section being studied—is known, it is possible to compute precisely the production levels required at every intermediate stage to supply the total sum of end products. Further, it is possible to determine the effect at every point in the production process of a specified change in the volume and mix of end products.

By means off input-output analysis, it is possible to determine the effects throughout the economy, for example, of a 10 per cent increase in automobile production. Solution of the mathematical matrix will give not only the increase in direct requirements for steel, power, glass, etc., but, more important, will also give secondary and tertiary effects, such as the added requirements for electric power by the glass makers, the steelmakers, the tire makers, etc., which result from the increase in automobile production.

Potential Use in Basic Industries

Its potential industrial value is largely limited to the analysis of basic industries whose products enter into many other products and, most important, where there is a high degree of circular flow. For example, coal enters into many other products, both as a fuel and as a source of chemicals. The impact on the coal industry of a 10 billion dollar highway program would be manifested in a variety of direct and indirect ways. The added cement requirements would produce a direct increase in coal requirements; the added steel requirements would also produce corresponding direct requirements for coking coal. But both the added cement requirements and the added steel requirements would produce added demands for electric power—which, in turn, would further increase coal requirements. The added coal requirement would itself result in further demands for more electric power, which, in turn, would raise the coal requirements again. The cement industry, the steel industry, the power industry, and the coal industry would require more capital equipment and again the steel and power requirements would be upped and the coal requirements further raised, and so on.

Thus the chain of reaction from the highway program goes on indefinitely—but progressively the indirect effects become less and less until the system reaches a new level of equilibrium. Clearly, if one were to try to compute the total increase in coal requirement which would flow from a 10 billion dollar highway program by trying to trace through these interactions, he would become hopelessly bogged down. Input-output analysis provides a theoretically exact and relatively easy (with a computer) solution to the problem.

Limitations of Input-output Analysis

The development of a matrix, however, is a major undertaking and probably would be quite costly. Its potential users are, therefore, limited to governments and to major corporations. It may be a useful tool in projecting long-range market forecasts and in ascertaining the effect on a given industry of changes elsewhere in the economy. It is also potentially useful in estimating the implications of technological change. It is of little value in short-run forecasting, where management decisions on inventories are not governed by the necessity to adhere to the fixed input-output ratios.

The real difficulties are that the raw data on individual input-output relationships are so difficult and costly to obtain that a major effort is involved. The Federal government has spent large sums collecting such information, and much of it is available to the public. But even so, the individual user would undoubtedly find that he needed much new information. A further difficulty is that even the largest tables of 400 separate industry categories require considerable aggregation. This is particularly true in dynamic industries such as electronics where, for example, the total copper used must be expressed as a fixed percentage of the dollar value of the combined industry output, regardless of the continuing changes in individual designs.

Mathematical Logic

Mathematical or symbolic logic is a method of logical reasoning in formal mathematical terms. As a language of reasoning, it has a great advantage over verbal language in that it is designed to be rigorously free of contradiction and ambiguity.

Symbols Expressing Logical Relations

Mathematical logic is concerned with fundamental logical relations. For example, assume that A and B are sets of individual objects, quantities, or characteristics. The fundamental relations between these sets can be expressed symbolically as:

$A \subset B$	Which means that all the items in group A are also contained in group B.
$A + B$	Which means "either A or B"
$A\ B$	Which means "both A and B"
A'	Which means "not A."

By use of these four fundamental expressions, very complex logical problems can be described and analyzed, just as the more familiar algebra provides a symbolic means of describing and solving complex numerical problems.

Probably the single most important application of mathematical logic so far has been in the design of the very complex circuitry of electronic computers. There has also been some experimental application of mathematical logic to contracts, legal codes, and similar complex documents, which are difficult if not impossible to trace through without an orderly method of logical analysis.

Application to Priority Problems

Mathematical logic can also be used in some cases to reduce the overwhelming complexity of some types of management problems. Although its uses so far have been limited, mathematical logic has application to a general class of problems of scheduling or sequencing individual operations in a production shop.

The potential importance of analytical techniques for solving problems of this type is illustrated by the fact that there are 207,360,000 different sequences possible for only five production jobs, each of which passes over the same four machines. Mathematical logic provides a means of quickly identifying those sequences that meet all the technical and priority conditions.

The problems of priorities in scheduling work are of many different types, and it is impossible to specify in advance the type of solution best suited to each. Problems may involve choosing among alternative sequences for individual steps. They may also involve machines that can be used alternatively on some parts and not on others. It seems probable that the methods developed by Akers and and Friedman * for scheduling production can be usefully applied to some types of problems, either for a complete solution, or to narrow greatly the number of alternatives to be considered.

In many problems involving priorities, it is important to invoke a high priority on individual components only in so far as it will actually contribute to the earlier completion of the total job. There is nothing to be gained by using a priority to advance the completion of a turbine rotor that cannot, in any case, be assembled and delivered until weeks later when a housing casting has been completed. In fact, if machining capacity can be used instead to produce a rotor for lower priority turbines that can be assembled and delivered at once, there is an actual loss in making the higher priority rotor first. Although unproven, mathematical logic appears to have potential application to this type of problem.

TRIAL-AND-ERROR SOLUTIONS BY SIMULATION

Simulation

Simulation is a systematic trial-and-error procedure for solving complex problems.

Many management problems are so complex that it is practically impossible to approximate the optimum by use of mathematical analysis. Under these circumstances, good answers often can be found by simulating the interaction of important elements of the problem. Simulation offers the possibility of comparing many alternative courses of action and of assessing the profit significance of events over which management will have no control.

In one sense, there is nothing new about simulation. A corporate planner who sets forth half a dozen alternative investment schemes, together with the costs and expected return from each, is simulating each alternative so that his board of directors can judge their relative merits. But the new mathematical tools, together with high-speed computers, extend tremendously the capabilities of simulation processes.

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* S. B. Akers, Jr., and J. Friedman, "A Non-numerical Approach to Production Scheduling Problems," *Journal of the Operations Research Society of America,* Vol. 3, No. 4, pp. 429-442. November 1955.

Example: Assessing Risks in a Construction Bid

An example may be the best way to illustrate the capabilities, as well as the limitations, of modern simulation technique. A construction firm is considering making a bid on a potentially highly profitable contract. The bid request stipulates a high penalty for completion later than a specified date. The penalty is such that a delay of a month or more will not only eliminate the expected profit on the job, but will seriously cut into the construction company's working capital. The president must assess the risks he would be running before he puts in a bid.

Simulation provides one means of assessing these risks. A first step is to list the various contingencies that may occur. The schedule for each phase of the job is subject to a large number of unpredictable occurrences. In some cases, these events may delay the job; in others, they may speed up the job and thus counteract the effects of other delays. In still other cases, a delay in one factor may make little or no difference in the final completion date because other delays have already occurred. For example, delivery of pipe a week late might, in one case, cause no additional delay because bad weather had already slowed down the job by two weeks. In another case, the same delay of pipe might slow down the job because the period of bad weather occurs after the delivery of pipe rather than before.

It is thus apparent that simply adding up the probable delays, and offsetting them against time saved in other phases of the job, will not suffice. Equally, an exact mathematical solution will probably be impossible because of the complexity of the problem, and because many of the individual factors will not vary according to simple mathematical formulas. In other words, the probabilities of different outcomes of an unpredictable factor cannot be described in mathematical expressions that can be solved easily. But a close estimate of the chances of late completion can be made by simulating the actual construction schedule under different combinations of the unpredictable factors.

A. *Defining variable factors:* Some of the more important factors that are subject to unpredictable variations are:

1. Weather
2. Strikes on the job
3. Strikes in suppliers' plants, such as cement and steel manufacturers

4. Other delays (or advances) in delivery of materials
5. Failure of one or more purchased materials to pass inspection after it has been delivered to the site
6. Unexpected trouble (or unexpected ease) in excavation due to unknown geological structures
7. Variations in actual work schedules from plan due to estimating errors
8. Variations from plan in the rate at which construction labor is recruited.

B. *Estimating probability:* A next task is to form some estimate of the distribution of the probabilities of different outcomes for each of these unpredictable events. The probability of bad weather on any day, or sequence of days, can best be ascertained by obtaining weather records for the last twenty or thirty years, or more if possible.

The probabilities of strikes on the job or in suppliers' plants probably have to be constructed in part from past records, and in part from the judgment of several informed people, such as labor reporters, mediators, and industrial relations supervisors. Obviously the time of contract renewal, economic conditions, and the attitude of both management and labor leadership will be important factors.

At this point it might be argued that this is nothing more than dressing up intuitive judgment, but the simulation process provides a way of fitting into the over-all solution, in a systematic way, the individual elements of judgment, as well as those factors, such as probability of rain, which can be measured by objective means.

The probability of inspection troubles, the probability of variation from plan in the work schedules, and variations in the dates of delivery of supplies can be estimated by examination of past records of the company in respect to these variances.

C. *Forecasting the effect of variables:* The next task is to estimate the effects of these events on the progress of construction. Take weather, for example. Examination of past rainfall records in relation to construction records on previous jobs might show that, during the excavation stage, rain had the following effects:

1/4″ to 1″	during any day would halt work for the day.
1″ to 2″	during any one- or two-day period would halt work for three days.
2″ to 4″	during any period of one to three days would halt work for five days . . . etc.

After excavation had been completed and footings poured, the effects of rain would have a different set of values; and after the structure was closed in, rain would have no significant effect.

A similar table covering the effects of frost would also be required. Temperature records would be converted to expected inches of frozen ground, It might be, for example, that according to the past records of the company, two to four inches of frost slows excavation by 10 per cent; four to ten inches results in a 30 per cent reduction; and, with more than ten inches, excavation work ceases.

D. *Defining manpower capacity and delivery requirements:* Next, a complete schedule of construction is prepared, showing the number of man-days and machine-days required for each phase of the job. Required delivery dates for materials and supplies are also specified, and lead-times on ordering are calculated.

E. *Simulating progress in construction:* The actual simulation can be done with clerks and desk calculators or with high-speed computers. The advantage of the computer is that, once the problem is set up and programed for the computer, a large number of runs can be made for very little additional cost. Assume that in this case a computer is used.

A first step is for the computer to select, at random, the weather records for one of the previous years. The computer does this in much the same way that one selects one slip of paper from a jar containing slips on which were written the weather records for all years for which records were available.

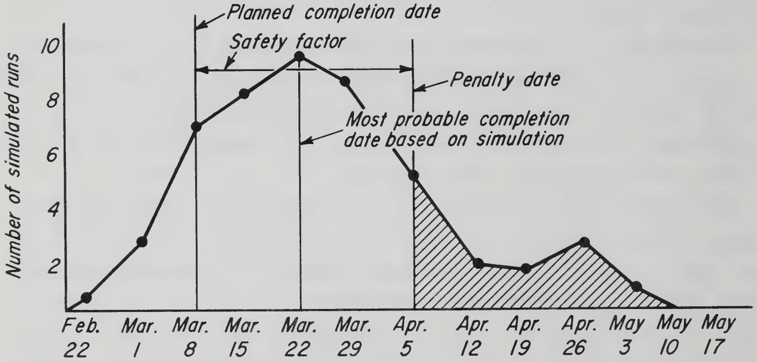
Orders for supplies are next "placed" on the days specified in the plans. Actual deliveries are calculated by taking the scheduled delivery time, and adding or subtracting the variation selected randomly from the probability distribution stored in the computer. In this case, the computer selects at random one value for each variation from a weighted distribution. For example, if the probability that a steel order will be delivered on time is 0.41 (meaning forty-one times out of a hundred it will be delivered on time), that it will be one week early is .05, and that it will be one week late is 0.15, the computer will do the equivalent of drawing at random one slip of paper from a jar in which there are 41 marked "on time," 5 marked "one week early," and 15 marked "one week late."

In a similar fashion, the computer will begin to calculate the successive phases of actual construction. If labor recruiting is slow, and if there is an excessive breakdown of earth-moving equipment, the excavation will be slowed down correspondingly. Similarly, wet weather, as indicated by the weather-record selector, will delay excavation.

If excavation is delayed into the winter season, it may be that an early frost will further slow the work. In contrast, an exceptionally late freezing of the ground will allow excavation delays to be made up for during the time the ground is normally frozen.

In this manner, the computer completes its simulation of the entire job—using both the initial planning figures and the modifications of these figures in accordance with randomly selected values representing the relative probabilities of alternative outcomes. The final product will be a simulated actual completion date. The entire process is now repeated many times. And, of course, each time specific values for each variable are reselected from the probability distributions. The results are next plotted, as shown in Figure 6, according to the degrees of variation between the simulated completion dates and the planned completion date.

Fig. 6



F. *Interpreting the results:* Figure 6 shows that in a total of 51 simulations runs, one completion was three weeks early, three were two weeks early, seven were one week early, etc.

On the basis of 51 runs, the following estimates can be made:

1. The probability of being ahead of schedule is 11/51 or 0.22
2. The probability of being on schedule is 9/51 or 0.17.
3. The probability of being behind schedule is 31/51 or 0.61
4. The most probable completion date is March 22
5. There is a probability of 8/53 or 0.15 that the job will not be completed until after the penalty date.

The simulation model can also be used to test the probable effects of other factors, such as strikes, which cannot be measured even as probabilities. For example, the probable effect of a possible strike in one of the suppliers' plants can be measured by changing the computer program to delay the delivery of the material by periods of varying lengths. By making several re-runs with each alternative strike assumption, one can form a good estimate of the probable effects of a strike of the assumed duration. Although the model cannot predict the probability of the strike, it can measure the probable effects of an assumed strike. In the light of these probable effects, management can, at this point, determine whether or not the dangers posed by the threat of a strike are sufficient to cause a change in its decision to submit a bid.

The contractor may conclude that the indicated risk of penalty is satisfactory—or that it is too great. A further calculation could be made, which would compare the probabilities and costs of late completion with the probabilities and profits of completion on or before the penalty date. But such a calculation would be an aid to judgment only. The contractor would also have to consider what intangible effect a failure to meet the penalty date would have on his reputation, on his cash position, and on his credit.

If he decided that the indicated risk was too great, he might next modify his simulation model by moving forward certain starting dates, by recruiting additional labor, by assigning additional equipment, or by stockpiling critical supplies. He could then re-run the simulation to see whether the risk had been brought down to an acceptable level.

Potential Uses of Simulation

There is a broad range of potential applications of simulation to management problems. These include:

1. Scheduling a large job shop
2. Scheduling the movement of ships, railway cars, trucks, aircraft, or combinations thereof
3. Planning for a new-model change-over on an assembly line
4. Developing long-range plans and checking their adequacy to meet a variety of alternative economic and market conditions
5. Testing the effectiveness of various alternative means of adjusting production and inventory levels to changes in sales
6. Evaluating the effect of time-lags in a reporting system on the decision-making process

7. Developing the optimum combination of distribution facilities and inventory policies

8. Finding the optimum program for maintenance of equipment

In the bidding example used above, time was the key factor being measured. It is equally possible to use simulation for evaluating results in terms of costs, efficiency, earnings, share of the market, or any other criterion or set of criteria. With simulation, it is possible to use several criteria simultaneously without having, at the outset, to relate them quantitatively.

Limitations of Simulation

The following are the principal limitations on the use of simulation:

1. It may not be possible to describe the interaction of the various parts of the problem with sufficient accuracy to make the answer significantly better than one derived from experienced judgment.

2. The costs of data collection, computer programing, and computer operation are substantial; and the potential saving must be sufficiently attractive to justify the expense. It seems almost certain, however, that as further experience is gained in computer simulation, computer programing techniques will be substantially improved; as a result, the costs of building simulation models should decline.*

3. Construction of the model takes time; therefore simulation cannot be applied to crash programs. On the other hand, once a model has been constructed, it often can be modified with relative ease and speed to solve problems created by new and unexpected situations.

Advantages of Simulation

The major advantages of simulation, when it is carried out with electronic computers, are:

1. The ability to handle uncertainty by means of probability analysis.

2. The ability to handle very complex problems and to find either the precisely or the approximately best answers.

3. The ability to analyze—at least to some extent—the implications of competitive situations and the relative merits of alternative strategies.

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* Since this was written H. M. Markewitz, of the RAND Corporation, has developed a special computer language for preparing computer programs for simulating problems.

4. The ability, through computers, to try out literally hundreds of alternative assumptions, strategies, and decision rules.

5. A limited ability to evaluate risk.

6. The ability to analyze dynamic situations in which each decision sets the conditions under which the following decisions must be made.

7. The ability to analyze the effects of time-lags in actual decision processes.

8. The ability to make "sensitivity analyses" by means of which one can determine whether a small change in a single factor will result in a major or a minor change in the outcome. If the former, the simulation model can show the degree to which the critical factor must be controlled to avoid violent fluctuations.

9. The ability to analyze quickly new and unexpected situations so that decisions to take advantage of opportunities can be made promptly.

Finally, the great advantage of simulation is that it is not restricted by the requirement that the model be solvable mathematically. It is this requirement that today places the ceiling on the usefulness of many of the strictly mathematical solutions to management problems —by forcing the analyst to depart from a realistic and accurate model in order to make a mathematical solution possible.

Monte Carlo Technique

The Monte Carlo technique is a form of simulation that provides experimental, as contrasted to theoretical, answers to problems involving the complex interaction of many random events. It often can be a useful element in a simulation solution.

Many management problems involve events whose occurrence can only be predicted as probabilities. These problems may be solved rigorously by mathematical means or, alternatively, by a form of simulation called the Monte Carlo technique. In developing the probability distribution of an event, which is a mathematical statement of the relative probabilities of each alternative outcome, a statistician will examine the available data on past occurrences of the event to see whether one of the standard probability-distribution curves can be used as an accurate approximation. In doing this, he will apply a variety of mathematical tests, from which he will conclude that a given mathematical expression can or cannot be used to represent the probabilities. If it cannot, he will have to use the nonstandard probability distribution as best he can. Usually this will mean that his calculations from here on will be considerably more tedious than if he had been able to use a standard formula.

Moreover, most management problems in which uncertainty is present involve the interaction of several random events, and it is necessary to calculate the probability of each of the different outcomes arising out of these interactions. It is in such problems that the Monte Carlo technique is useful. An example of this kind of problem can be found in the sugar cane industry.

Assessing Uncertainty by Theoretical Methods

A grower must decide how much cane he will plant each year in order to meet a fixed quota of sugar deliveries. Unlike most crops, sugar cane takes two full years to mature; and, as there are substantial variations between planned and actual production owing to differences in rainfall and temperature from year to year, a safety stock is carried over from year to year in order to meet the fixed delivery schedules. This carry-over stock is also required to protect deliveries in the event that the start of the new harvesting season is delayed by bad weather.

Thus the decision on what acreage to plant in 1957 for harvesting in 1959 depends on the interaction of four unpredictable factors: the difference between estimated and actual yield of the cane planted in 1955 and harvested in 1957; the difference between estimated and actual yield of the cane planted in 1956 for harvest in 1958; the variation from estimate in the 1959 harvest of cane to be planted in 1957; and finally the probability that the start of the 1960 harvest will be delayed by weather.

The first step in the solution of this problem by mathematical means would be to study past records to see what the nature of each of the variances might be. If it appeared that they could be represented by standard equations, the combining of the individual probability distributions could be handled mathematically, to provide the probability for each alternative outcome in 1960. The answer might be stated as follows: For a specific acreage planted in 1957, and taking into consideration the probabilities of variances in the yields of 1957, 1958, and 1959, plus the probability of a late season in 1960:

The probability of a 20,000-30,000 ton overage is 3%.
The probability of a 10,000-20,000 ton overage is 15%.
The probability of a 0-10,000 ton overage is 25%.

The probability of a 0-10,000 ton shortage is 30%.
The probability of a 10,000-20,000 ton shortage is 20%.
The probability of a 20,000-30,000 ton shortage is 7%.

With this information, and with information on the costs of carrying an overage into the next year, as well as the costs of failing to meet the quota, it is possible to determine that acreage to plant in 1957 which will minimize the total expected costs arising out of the variation of each of the harvests from plan.

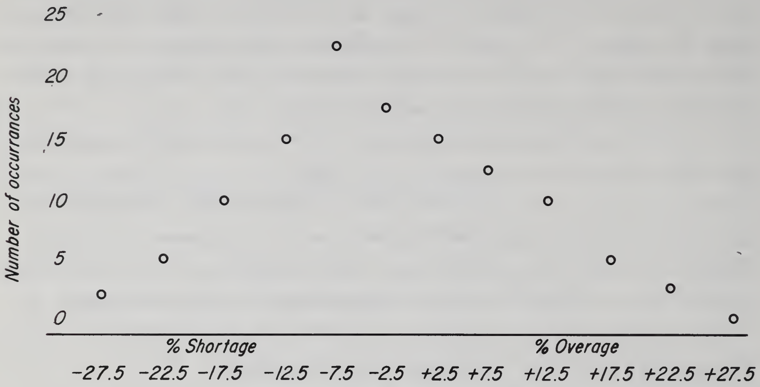
If, in this problem, the statistical analyst concluded that he could not safely represent the actual probabilities by one of the standard distributions, he would be faced with a substantially increased computational problem. Provided the number of alternatives considered for each variable were sufficient to yield a usable answer, something in the order of 250,000 separate calculations might be required.

Assessing Uncertainty by Monte Carlo Simulation

Monte Carlo techniques would offer the opportunity to greatly reduce the computational work in solving this crop problem, without significantly detracting from the validity of the solution. If the Monte Carlo technique is applied in this case, four groups of 100 cards each are prepared representing each of the four probability distributions. For example, if the probability is 3 per cent that the 1957 production will be between 20,000 and 25,000 tons short of plan, and 10 per cent that it will be between 5,000 and 10,000 tons short, three cards marked —22,500 (the mid-point of the first class) and ten cards marked —7,500 (the mid-point of the second class) are put in a deck together with cards representing all the other possibilities for the 1957 season. Each of the four decks is thus made up of cards representing the relative probability of each of the possible outcomes for each year. With the decks, after shuffling, representing each of the four years face down, one draws one card at random from each, and adds up the total of the four cards drawn. The result might be this:

	Card drawn	
1957 season	7,500	
1958 season	—12,500	(shortage)
1959 season	2,500	
1960 season	7,500	
Total	5,000	(overage)

This process, which simulates a single possible sequence of seasons, is repeated until a sufficient record is built up to give a good approximation of the relative probability of each total outcome. The plotted results look like this:



A curve is next drawn to smooth out the individual points. The relative probability of each degree of shortage or overage is now found by measuring the height of the curve at that point. As before, the optimum acreage to plant will be that acreage which will minimize the probable costs of overage and shortage in 1960.

A final part of the problem would be to estimate the number of runs necessary to give the degree of reliability required of the answer. Although techniques now exist for such estimates, further research work probably is needed to find more efficient ways of laying out the sequence of runs, and of evaluating the probable error in the results.

If the problem were big enough, or if it were part of a larger problem, it would probably pay to use a computer. In this case, the computer would, in essence, duplicate the random drawing of cards as outlined above, except that it would operate thousands of times faster than would a human being.

Advantages and Limitations of Monte Carlo Technique

The Monte Carlo technique can be applied to many types of problems. Complicated queuing or waiting-line problems often can be solved more easily by this means than by mathematical means. Transportation, production, inventory, and distribution problems usually have random factors that are too complicated for formal mathematical treatment, even when the individual probabilities can be adequately represented by standard probability equations.

One possible disadvantage of the Monte Carlo method is that its accuracy is limited, by practical considerations, to two places. This disadvantage is more theoretical than practical, however, because in most management problems this degree of accuracy is not only sufficient, but is equal to the accuracy of at least some of the original data.

Operational Gaming

In essence, operational gaming is a form of simulation in which human decision makers, instead of computers, are used to simulate at least some of the critical elements of the problem under study.

The application of operational gaming to management decision-making problems is an outgrowth of the development of war gaming by the military services. During the past ten years military research organizations, such as the RAND Corporation (Air Force) and the Operations Research Office (Army), have put substantial resources in manpower and money into the development of military games. These games are highly realistic and reproduce a large number of factors that would enter into actual military decisions.

Much of this experience has practical application to the reconstruction of business situations. The General Electric Company, the management consulting firms of McKinsey & Company, Inc., and Booz, Allen & Hamilton, and the American Management Association, among others, are experimenting with games that simulate different types of competitive business situations. The AMA work is sufficiently advanced to demonstrate that games can be constructed to simulate decision-making processes in competitive business situations.

The AMA game is designed to stimulate and improve decision-making skills of the individual executive. The General Electric game is designed to demonstrate to middle levels of production management the advantages of new mathematical decision rules developed to improve production scheduling. The McKinsey work is directed primarily towards the problems of the organization of management to compete more effectively and to adapt more rapidly to new situations.

Elements of a Typical Game

In the AMA game, two or more teams, representing separate firms, compete in the same market. At the beginning of each game, each team is given equal assets in cash, productive capacity, and inventory, and a starting share of the market. Each team is then required to make a series of decisions covering the next operational period of three months. These decisions include:

1. Quantity of product to be manufactured during period
2. Plant expansion (if any) to be started during period
3. Marketing expenditures
4. Research and development expenditures
5. Price to be set on product
6. Excess capacity, if any, to be liquidated (at a fraction of initial cost)
7. Market information to be purchased (here a half dozen items, such as competitors' expenditures on research and development, are available for a price)

The decisions made by each of the teams are next given to referees, who then determine the number of units actually sold by each team during the period. AMA uses a computer that has been previously programed to simulate the market. The nature of the market is specified by the rules of the game so that for any set of price, production, and marketing decisions by the teams, both the total sales during the period and the sales of each competitor can be calculated. The players, obviously, are not told the exact nature of these rules for determining total market and shares of the market, but must deduce them from experience in playing. The game rules also provide that the costs to a team of increasing its share of the market, in terms both of price concessions and marketing costs, will, at some point, begin to rise much more rapidly than will the resulting increase in sales income. For example, if the costs of increasing one's share of the market by 2 per cent were $300,000, the cost of a 4 per cent increase might be $1,200,000.

In addition, certain constraints are placed on the range of possible decisions. For example, the rate of investment in new plants cannot be changed more than plus or minus 15 per cent from one quarter to the next. Production, insofar as it is below capacity, cannot be increased more than 4 per cent a quarter, nor can it be reduced more than 10 per cent a quarter. Price cannot be varied more than 10 per cent a quarter. The purpose of these limitations is to prevent the teams from making unrealistically violent shifts in operating levels.

When the sales of each team have been determined, a simplified statement for each can be prepared which will show cash, inventory, and plant as of the end of the period. This statement then becomes the basis for the next round of decisions by the players. The game may be continued for any number of periods.

In order to simulate actual conditions, the market used in the game may have a long-term growth trend, a mid-term cyclical pattern, and short-term fluctuations of purely random character. As in real life, some combinations of production, price, and marketing policies will produce greater earnings than others. And as in real life, earnings will depend in part on the decisions of competitors, in part on the market structure, and in part on the internal costs of the business. The skillful team, like skillful management, will make its decisions in light of these factors in a way which will maximize long-run profits.

As in real life, there need be no agreed goals by which the winners and losers will be measured. The process of playing itself will help players to think through more carefully the goals they seek. Share of the market, total assets, and return on investment will, of course, all be measures of relative success or failure.

Gearing Games to Specific Objectives

The AMA game in its present form has demonstrated the feasibility of gaming and its potential as a training tool. Much work still needs to be done, however, so that one can be reasonably sure that a game is in fact accomplishing the purposes for which it is designed. A game may emphasize the external strategy of a company in competition for markets, or it may emphasize the internal problems of organization, or both. Individual games can be developed to emphasize one or more of a variety of problems, such as long-range planning, finance, competive pricing, marketing, and research and development.

Also, once a basic game has been set up, it can be altered from time to time to emphasize particular activities and to reduce the significance of others. For example, a game might emphasize financial factors, such as taxes, depletion allowances, equity financing, interest rates, etc. While no game can duplicate differences in the abilities of individuals, such as research scientists, it can treat the development of new products as unpredictable random events for which the probability of occurrence can be increased by increasing the research budget, or by shifting the direction of research programs, and by raising the salary levels and competence of researchers.

Further, it may be possible to modify a game by replacing one or more of the teams with staff players, whose objective is not to win but to creat a variety of situations, such as price-cuting or credit expansion. This would give other players experience in coping with specific problems that might or might not arise during the course of games played entirely by competive teams.

Advantages of Gaming

The more immediate use of gaming lies in the field of executive development and training. Because games are so absorbing, and because the lessons learned penetrate deeeply into human consciousness, games appear to be an especially effective and efficient learning tool. They are also so stimulating that most executives are eager to continue to play.

Among the benefits players can gain are:

A. *Training in planning:* Because gaming involves a long series of decisions in which the player must continue to live with his past decisions, and because time is greatly foreshortened, it is possible for the player to learn the values of planning as well as the penalties for operating on a day-to-day basis. For example, the team that fails to maintain an adequate research and development budget will find its market position slipping over successive periods of play, even though it is maintaining a high rate of marketing expenditure. And even when the team realizes the cause of its difficulty, it will have to live with the problem until the results of increased research and development expenditures begin to halt the decline in its share of the market. During this period the plant will be operating at a fraction of capacity, and cash undoubtedly will be short. The team will thus be forced to live within its resources.

B. *Training in analysis of past experience:* Games may be designed to be played most successfully by those who are able to analyze past experience in order to find the critical relationships among the many factors involved. For example, skillful use of marketing information, together with a limited degree of experimentation in altering price and marketing expenditures, can reveal the point at which a further decrease in prices or further increase in marketing expenditures will cease to increase earnings.

C. *Training in recognizing the many factors that normally affect the success or failure of management:* Often managers tend to make all their decisions on the basis of three or four favorite factors or indicators, and thus tend to neglect other important factors. Gaming demonstrates the penalties for failure to recognize important factors as well as for failure to evaluate their significance.

D. *Training in phases of business with which the executive may be quite unfamiliar:* For example, a middle-level executive who is in line for a general-management responsibility may never have been outside the production department. Gaming can offer an effective way to provide, to some degree, simulated experience in finance, sales, market research, etc. Similarly, gaming can be used to increase the appreciation of executives in one department of the importance of the work in other departments.

In addition, games can be designed that will stress the problems of internal organization. Because of the pressures of the game on the team to make good decisions in a limited time, the team will be forced to organize itself effectively if it is to win. Because of the greatly foreshortened time scale, a team may go through several different periods in which the marketing and competitive conditions change significantly. Under such conditions the team, to be successful, must be able to modify its organization rapidly to meet each new set of conditions.

Finally, while games cannot be used directly as a means of developing an optimum long-range plan, they may be helpful in gaining insights into problems that lie ahead. For example, many industries, such as television manufacturing, pass through the successive states of rapid expansion of markets; a subsequent leveling off of the market accompanied by overproduction; a period of shakeout accompanied by failures, mergers, and integration of production; and, finally, maturity for those who survive. During the initial period, when markets are growing rapidly, the company executives will be aggressive. They will concentrate on rapid expansion of production, expansion into new markets and areas—financing the expansion any way they can. They will tend to regard costs as secondary, and many will probably regard continuing rapid expansion as an unchanging way of life.

In such situations, operational gaming may become a very valuable tool to provide vicariously the shocks and lessons of overexpansion, of excessive costs and unsound financing, and finally of the shakeout. A game could be constructed in which the teams were faced with the temptations of a seemingly endlessly expanding market and in which they subsequently would, as a result of their own past decisions, be faced with reduced profit margins, overcapacity, and high fixed costs, and would have to live with their mistakes. In general, operational gaming is potentially a useful device for making executives aware of the possible ultimate implications of past decisions that cannot be undone.

Limitations of Gaming

Gaming has at least two potential weaknesses:

1. The necessary compression of time may warp the game as a reflection of reality. For example, when decisions that are usually made over a period of three months are compressed into fifteen minutes, the decision maker no longer has the time for considered analysis of his problem. This shortcoming probably can be alleviated by spacing the periods of play over a period of days or weeks so that only a few minutes a day (or half a day each week) are devoted to the game.
2. Players may attempt to transfer directly to their business those specific decision-making strategies they have found to be successful in a game. At this stage it is probably impossible to devise a game so close to reality, yet simple enough to play, that a successful game strategy can be transferred unaltered to actual problems. But this danger can be overcome by deliberately adjusting the game from one play to the next so that the player is confronted with constantly changing circumstances that he must correctly diagnose and react to.

The game probably cannot reveal anything that has not consciously been built into it. Hence it may turn out that the experience of constructing a game is the most valuable product. For, if a game is patterned on a specific industry, then the nature of the industry, its markets, and its relation to the general economy, as well as the company's internal cost structure, must be built into the game. Thus the builder of the game must, necessarily, incorporate the key factors of success in his industry if the game is to be useful.

In summary, operational gaming must be viewed as still in the experimental stage. It is easy enough to construct a management game that is fascinating to play, but it is considerably more difficult to construct a game in which the experiences gained in playing will contribute to the ability of executives to manage their corporations more successfully. Corporations with substantial research programs should find in operational gaming an area of research that gives the possibility of a return at least comparable to the return on their investment for product research and development. But it is important that companies going into gaming regard it as research, with all the attendant risks of failure, and not yet as a proven tool of management.

Systems Analysis

A major objective in the use of mathematical analysis is the application of its many techniques to the over-all problems of management. Such application of analytic tools to broad technical and organizational systems has come to be termed "systems analysis."

A "system," in this context, can be defined as any complex of interrelated activities that must be closely coordinated if efficiency is to be realized. The production, refining, transportation, storage, and distribution activities of a major petroleum company, for example, constitute such a system, because all these activities must be run as a single system if the company as a whole is to operate most efficiently. McKinsey & Company, Inc., has recently made a major study of a system in which raw materials are reproduced at several locations abroad, partially refined at foreign ports, stored, shipped to mainland refineries, stored again, refined, and finally distributed. In this case, the components of the system are so directly related to one another that a reduction in the operating costs of one activity often raises costs of other activities. It was therefore the purpose of his study to optimize the over-all operating efficiency in such a way that maximum total savings were realized, even if this resulted in increased costs on some of the components.

If an analytical model or framework can be constructed that embraces most of the principal physical and financial elements of corporate activity, it is possible in many cases to approximate closely the best combination of all of these factors affecting the success of the enterprise. These include, for example, price policy, production and inventory programs, distribution channels, marketing programs, investment programs, and financial plans.

Difficulties of Simulating a Total System

The realization of such goals, however, will be conditioned by several difficulties, although it should be possible to surmount these obstacles to the degree necessary to produce practical results. Some difficulties that will be encountered in building a workable model of the enterprise include the following:

1. Mathematical models are at best imperfect abstracts from real-

ity. Not only must the quantifiable relationships be simplified to be manageable, but those elements of the problem that cannot be quantified are eliminated entirely. The important influences of human creativity and human judgment cannot be built into a model.

2. Enlargement of the scope of the problem increases the number of variables fantastically. For example, a problem having only ten variables, each of which has only ten possible values, produces a total of 10^{10} or 10,000,000,000 separate situations to evaluate and compare. Actual problems of top management contain billions, if not decillions, of alternative possibilities.

In part, this problem of complexity is being solved by the mathematicians who are developing mathematical shortcuts, such as linear programing, by means of which the sequential analysis of all possible combinations can be drastically reduced. But the remainder of the problem must still be reduced to manageable size.

This can be done in two ways, both of which require high orders of skill and judgment. First, those factors that are not "important" to the problem being studied can be dropped out. Second, by aggregation the effects of several factors can be lumped together and represented by a single composite factor. Both of these processes are difficult and, if incorrectly done, can lead to very bad results.*

3. No model can be more accurate than its underlying assumptions. The more complicated the model, the greater the risk of error —simply because the effect of a single assumption is less easily discernible in the result.

4. The selection of criteria for judging alternatives becomes increasingly difficult as the size of the problem increases. The criterion for a warehousing problem may be relatively easy to define as minimizing of the total costs of storage, transportation, and lost sales. The criterion for judging a long-range plan is hard to define. Is it share of the market, growth of assets, stability of earnings, ability to withstand a depression—or something else? Is a bold or a conservative course the better?

Advantages of Systems Analysis

In spite of these difficulties—most of which arise in any method of analysis—systems analysis can be a valuable tool for planning both long-range action and current operations. By defining the implications and probable outcome of each alternative course of action, systems analysis enables the executive to look at the important alternatives—and their implications—and thus to make judgments as free as possible of guesswork.

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* See Charles Hitch, the RAND Corporation, Report P-699, 20 July, 1955.

A systems analysis embracing all the principal activities of the enterprise, or a major part thereof, serves also to demonstrate the interrelation of these activities. A proposal to improve the efficiency of one activity can be judged in the context of the corporation as a whole. For example, will a proposal to cut inventories result in a net saving, or will there be added costs of shorter production runs, plus costs of poorer customer service, that outweigh the inventory savings?

COMPUTERS AS A TOOL IN SOLVING THESE PROBLEMS

Electronic computation is often thought of as being synonymous with mathematical analysis. Actually, a computer is simply the fantastically capable tool of a skilled problem solver. The great advantage of the computer is not that it is able to perform 100,000 additions or 20,000 multiplications a second, or that it can replace the trained analyst—but that it can be instructed to perform a long sequence of operations without intervention from the human operator.

If one were using a desk calculator to do the following problem, it would be necessary to punch in separately each number, to record each intermediate result on a scratch pad, and subsequently to repunch these intermediate numbers back into the machine.

1. Multiply	2,579,803		
by	57,607		
Record the answer		(a)	148,614,711,421
2. Multiply	1,760,119		
by	691,001		
3. Record the answer		(b)	1,216,243,989,119
4. Compare the two answers (a) and (b) and take the larger (b)		(c)	1,216,243,989,119
5. Subtract from (c)	50,169		
Record the answer		(d)	1,216,243,938,950
6. Take the square of	4,310		
Record the answer		(e)	18,576,100
7. Add (d) and (e)			
Record the answer			1,216,262,515,050

The numerical solutions of mathematical problems often require thousands of such individual steps and, if a solution is being found by successive approximations, often require that entire sequences of steps be repeated hundreds of times in order to arrive at an acceptable approximation of the true answer.

Such processes, if done by clerk and calculator, are extremely difficult, both because of the months of clerical time required and because of the errors which inevitably would creep in. A computer, on the other hand, can handle the entire operation from start to finish, with no help from the operator on intermediate steps.

Limitations and Advantages

The computer cannot, by itself, solve a problem—any more than a number-four iron can make a hole-in-one. The significance of computers to management is that they provide possibilities of solving problems that are too complex to solve by any other means. The great speed of the computer makes it possible to make a computation in a few seconds or minutes, and then, after the result has been studied, to repeat the process under different assumptions as many times as is necessary to arrive at, or close to, an optimum.

The very existence of computers has stimulated mathematicians to find solutions to problems that, in the past, were not even considered because of the impossibility of performing the necessary numerical calculations. For example, complex production and assembly operations, such as those found in the aircraft industry, can be brought under considerably more effective control by the use of computers than is otherwise possible. A major reason is that the new techniques of mathematical analysis permit management to incorporate directly into an electronic data-processing system many of the decision-making processes. For example, inventory levels can be adjusted automatically to conform to changes in demand, in warehouse space, and in production levels. Management is thus freed to concentrate on more basic problems which cannot be so handled.

In summary, computers are the tools that make today's techniques of mathematical analysis practical. They are almost always required for the solution of problems by simulation. And they make possible the solution of problems by such techniques as linear programing, which are usually prohibitively expensive to solve by other means.

Only after identifying the problem, and outlining the form of solution sought, however, should one decide whether to use a computer, punch cards, or a scratch pad. Naturally, the form of the solution will be conditioned by the means used. The decision on the tools to use will represent a balance between (a) the potential contributions to profits of each alternative approach and (b) the capabilities and costs of the equipment available.

Potential Developments

Probably as great strides in the computer field will be made in the next ten years as have been made in the last ten. Even the largest computers are now being found to be much too limited to handle many problems. Already prototype computers are being made with capacities a hundred times greater than present machines.

Herbert Simon of Carnegie Institute of Technology and Allen Newell of the RAND Corporation have embarked on a program of research in which a computer is being used to do things which heretofore have been regarded as intuitive processes. They have, so far, succeeded in programing a computer to re-prove the basic logical theorems of Newton, using only four fundamental axioms as a beginning. This initial success has led Simon to predict that in ten years computers may be doing many things which are now regarded as essentially intuitive or judgmental.

The central problem in expanding the use of computers is that of "language." Language not in the usual sense, but a new type of language which can be used to describe very accurately the logical nature of the problem under study and to translate this problem into the language of the computer. Everyone who has had experience, even indirectly, in the programing of computers has been struck, first, by the necessity to understand the problem with a precision never before even imagined and, second, by the very great labor involved in translating the problem into computer programs or language.

Further progress in our ability to apply computers to management problems will depend primarily on the progress made in design of a means of communication by which management-decision processes can be both accurately described and readily translated to computer language. The lack of such a language today does not preclude the application of computers to management problems, but it does make the process a difficult, costly, and time-consuming one.

Part III.

APPLICATIONS OF MATHEMATICAL ANALYSIS TO MANAGEMENT PROBLEMS

There have been applications of the newer decision-making techniques to most of the fields of management. But it is probably fair to say that these applications have so far only scratched the surface. Hence the record of successful applications is still very limited.

It is important to realize that at this stage each application must be tailored to the problem. Seeming similarities in problems cannot be taken uncritically as proof that, if one can be solved, the other can be solved by the same means. At the same time, many problems that appear to be quite different actually have a common theoretical structure. This makes it possible to use the basic elements of a solution developed for one problem in the solution of others.

A brief description of some applications that have been made—as well as some potential applications—may give the reader an insight into the uses and limitations of these analytic tools. These applications are discussed under three headings: "Over-all Planning and Control," "Production and Related Functions," and "Marketing."

Financial Planning

There are several important potential uses of mathematical tools for financial planning.

In most situations, the problem is not to find the investment program that will yield the highest return on unlimited funds, but rather to find the most productive uses of quite limited capital funds. This cannot be done if individual capital expenditure programs are considered piecemeal, for then they are judged, not against each other, but only against a given minimum return on investment. What is sought is the combination of expenditures that will return the greatest profit.

Example: Linear Programing for an Investment in an Electric Power System

Linear programing has been applied by the French mathematicians Masse and Gibrat to an investment of limited capital in new electrical generating capacity in France.* In the power industry, there are two related requirements of output: a capacity to meet peak loads and an ability to deliver total requirements. Each type of generating station has different cost characteristics in relation to those two requirements. In general, plants with high capital costs and low fuel and other operating costs are more suitable for supplying constant loads; and plants with low capital costs and high variable costs are more suitable for handling peak loads.

Masse and Gibrat found linear programing a useful tool in finding the mix of steam plants, hydroelectric plants (some with and some without reservoirs), and tidal plants that would provide both the necessary peak-output capacity and annual output for the minimum total cost.

Factors to Consider in Financial Plans

In most situations, financial planning involves a large number of interacting factors which make the evaluation of alternative plans and the selection of the optimum program an extremely difficult affair. Because of this complexity, mathematical analysis may prove to be of particular value in the solution of practical problems. Among the factors to consider, these are especially important:

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* P. Masse and R. Gibrat, "Applications of Linear Programming to Investments in the Electric Power Industry," *Management Science*, Vol. 3, No. 2, Jan. 1957.

1. Financial planning, like most planning, involves a substantial element of uncertainty: uncertainty as to sales potential, raw material and labor costs, price levels, product superiority, success of research and development, and strength of competition. (Probability theory is a useful tool for handling and evaluating such uncertainty.)
2. Federal, state, and local tax structures increasingly bear on the relative profitability of alternative investment opportunities. Depreciation schedules, abandonment losses, corporate income taxes, and property taxes all significantly affect the relative advantages of alternative plans.
3. Competing capital requirements often cannot be judged apart from each other. Investment in expanding capacity for producing present products may reduce working capital requirements because of a reduced need to carry seasonal inventories; while investment in capacity for producing new products may increase the requirements for working capital correspondingly.
4. The many alternative schedules for new plant construction over the years ahead add to the complications of analysis. If fewer but larger plants are built over the years, both capital costs and operating costs per unit of capacity will be decreased, but plant utilization immediately after construction may be low. If, instead, smaller plants are built more frequently, capital costs are higher, but utilization probably will be greater and distribution costs may be less.
5. The many alternative sources, and combinations of sources, of capital depend in part upon the nature of the proposed investment program, while the investment program is often conditioned by the sources of capital available. The financial plan must provide the most advantageous program for raising capital from among the various alternatives. These alternatives will include retained earnings, short-term borrowings, long-term borrowings, lease-back arrangements, and new equity capital. Not only are these interrelated, but their relative availability is in turn related to the use to which they are to be put and to the past record of the company.

While techniques such as linear programing, quadratic and dynamic programing, and queuing theory will have specific applications to parts of the over-all program, it appears doubtful that the over-all problem, if it is relatively complex, can be handled by formal solutions. Instead, simulation of financial plans, usually by use of computers, appears to be the only means available today by which all of the key factors can be handled simultaneously. In other words, there appears to be no possibility of solving mathematically for the single best plan. Simulation will, however, aid planners by providing the means to compare rapidly a large number of alternative combinations of factors, some under the control of management and some not.

Steps in Simulating a Financial Plan

It should be possible to program an entire financial plan so that it can be simulated on an electronic computer in a very few minutes, though the programing for simulation may take months. In such a simulation, the following steps would be taken:

1. A variety of alternative sales forecasts would be tested by successive runs.
2. Costs of production and of sales under a variety of assumptions, and for different sales volume, would be computed within the computer and used as the basis for further calculations.
3. The specific financial plan to be tested would be set up in the computer and the flow of funds programed according to plan.
4. Cash flow would be simulated from the income generated by sales; the costs of production and of sales, and by the allocation of available cash to liquid reserves, inventories, consumer credit, research and development, capital purchases, and dividends.

The net result would be the simulation of the key elements of the financial plan under a variety of assumptions in order to test the relative value of many different programs under a number of favorable and unfavorable circumstances. A given financial plan may be ideal for the level of sales projected by the economists over the next ten years. But if sales fall off significantly, or if competition drives down the profit margin—is the plan still feasible? A variety of other questions might be answered by re-runs of the simulation model:

1. What are the implications of changes in interest rates?
2. What are the implications of increases in wages?
3. How much of a reduction in sales revenues is possible under the plan being tested?
4. Does the plan provide flexibility to increase the planned rate of expansion?

5. How vulnerable is the plan to changes in the market price of the company's securities?

6. Can new opportunities, such as the chance to buy smaller businesses, be taken advantage of?

7. Does the plan provide adequate capital under a variety of circumstances?

8. What is the effect of variations in sales levels?

Since the criteria for judging the answers to such questions are extremely difficult to formulate apart from specific situations, it is almost certainly impossible to set up an economic model which can be used to find the "probably best" solution. For example, suppose two plans are considered: one offers the greater returns under a continually expanding market; the other provides lower fair-weather earnings, but is much more stable under unfavorable conditions. Which is the better? Only management can answer this problem, and then only if the actual alternatives are posed quantitatively. Simulation provides management with the tool to do this.

The role of the computer is that of a high-grade clerk that practically never makes mistakes. If it is feasible to make the many runs required without a computer, there is, of course, no reason for using one. The normal difficulty, however, is that by the time two or three alternatives have been calculated manually, the time available for deciding has been used up, and no further runs are made.

Accounting and Cost Analysis

Mathematical analysis offers the possibility of both improving the quality of accounting information and reducing the costs of accounting.

Sampling to Reduce Accounting Costs

Modern techniques of statistical sampling have made it possible to use relatively small samples to obtain surprisingly accurate approximations of the totals being sampled. For example, three major airlines are now settling their interline accounts on a sampling basis. Warren Alberts of United Airlines reports that on $4 million worth of billings a small test sample showed a variation of only 0.1 per cent, or $1,600, between 100 per cent accounting and the estimate based on the sample.

Sampling has long been used by industry as a means of controlling product quality. It is also fully accepted as a technique of auditing.

And it may become equally significant as a tool for controlling costs. By the use of modern mathematical techniques, it is often possible to design sampling procedures which, over the long run, will give expected errors of less than 1 per cent from samples that are a small fraction of the total. Devices such as stratified sampling, for example, make it possible to eliminate entirely the "one-in-a-million" chance that the sample will hit nothing but extreme values and thereby give grossly inaccurate results.

Further, sampling procedures can be designed to give any predetermined degree of accuracy, so it is possible to balance the cost of sampling against the costs of probable variations of the sample from the true value. Each proposed use of sampling should first be examined against present costs, however, to see whether the expected savings are sufficient to justify the change in procedures.

Sampling should have increasing application in internal cost accounting. Since internal cost accounting is used primarily for control purposes, its accuracy need only be at a level which is significant for control. The costs of fuel and power, the costs of maintenance and setup, in-process inventory costs, machine-center costs, and many others can be collected by sampling and analyzed statistically.

In the area of external accounting, where actual cash payments and receipts must be determined, greater accuracy is needed. It is only natural that management should be hesitant about using sampling in this area. Nevertheless, there have been at least two major applications of sampling of external accounting. In addition to the use of sampling by the airlines (mentioned above), the Railway Express Agency is using sampling to allocate railway express revenues among the railways on the basis of volume and mileage.

Linear Programing for Better Cost Information

Under certain circumstances linear programing can also become a useful cost analysis tool. When a complex production plan is revised —owing to the addition or cancellation of an order, or a change in the cost or availabality of a raw material—linear programing can be used to find the true incremental costs of the revision.

For example, a paper manufacturer may be able to obtain a large new contract for a certain kind of paper stock. Assuming he has already developed a linear program to find the optimum production schedule to meet existing orders, he can now add the new requirement to his program and obtain a revised optimum over-all schedule. This may well involve the shifting of one or more other orders to less efficient machines not now being used; it may also involve shifting some orders to other mills which are further away from the customers. It may at the same time provide much better over-all machine loading. The new solution of the linear programing matrix will not only provide the best over-all revised production plan, but will also provide an accurate measure of the net added costs resulting from these shifts. Thus management can make an accurate estimate of the total out-of-pocket costs involved in accepting a new order and can use these estimates in determining the actual effect on profits of a specific price quoted.

In the event that management must also consider other factors, such as customer relations, which are not included in the linear programing matrix, it may not be desirable to use that schedule which shows minimum costs. By use of linear programing, it is possible to determine the true added costs of extra customer service, or of nonstandard products manufactured to provide better service to large accounts.

Potential Extension of Cost Analysis

Normally, accounting is concerned only with events that have happened; it does not consider what has *not* happened—for example, it ignores the cost of lost sales. Mathematical analysis may make it possible to include such costs in accounting practices.

The advantages of analyzing the cost of lost sales * are plain, for this cost is an important factor to be considered in decisions on production, distribution, and marketing. In some cases the cost of lost sales arising from failure to maintain stocks at the consumer level can be estimated fairly precisely. It represents the sales not made during an outage period, plus the additional lost sales resulting from a loss in share of the market—which continues after supplies have been restored. In other cases the cost of lost sales can only be taken into consideration by setting up a series of alternative values for the cost of lost sales. If management believes that the cost is most likely to be represented by one of these alternatives, it will make decisions on production and inventories accordingly.

* This cost is discussed further under "Inventory Control" in the section on "Marketing."

Production

Both mathematical analysis and simulation have been successfully applied to several major aspects of production planning. In some cases, rigorous mathematical solutions are possible. In other cases, simulation has proved to be an effective tool.

Setting Over-all Production Levels

The concept of probability is perhaps one of the most important elements of mathematical analysis that has been introduced into production planning. The Plastics Division of Monsanto Chemical Company has developed a "probability budget," in which forecasts of sales are, for each period, expressed in probability distributions rather than as single numbers. Planning decisions are made on the basis of balancing the probable costs of under-production against the probable costs of over-production. As a result, the optimum production level may turn out to be higher or lower than the most probable costs of over-production. As a result, the optimum production level may turn out to be higher or lower than the most probable sales level, depending on whether the costs of lost sales are greater or less than the costs of over-production.

Problems in Planning Batch-type Production

The principal problem of production planning for job or batch-type products is to determine the quantities of each part to be produced, the machines to be used, the production schedules to be followed, and the sequence of parts to be processed at each machine station. The problem is compounded by the large number of interacting queuing situations throughout the job shop. Random fluctuations in processing times at critical stations may have far-reaching impact on schedules and performance in other areas of the shop. Full utilization of equipment and efficient schedule performance (small amount of delay) tend to be incompatible objectives. In large plants with hundreds of machines, the problem becomes extremely complex and the potential savings are substantial. As a result, considerable effort has been made to apply the techniques of mathematical analysis to finding the optimum pattern of production. Formulas involving only setup and inventory carrying costs have been used for years in calculating economic lot sizes. In all but a few cases, however, there are several other factors that influence significantly the optimum routing and scheduling of production.

A. *Factors to consider:*

1. *The over-all efficiency of machine loading.* For each specific mix of orders there will be one loading schedule that will get the most efficient use out of all available machines.

2. *Sequence of operations.* A schedule that theoretically provides forty hours of work for each machine in a given forty-hour week is of no value if the parts to be processed by some of the machines are not available from preceding machine stations until the middle of the week.

3. *Delivery dates, assembly schedules, and priorities.* Optimum schedules must take into account either fixed delivery schedules or delivery priorities. The best solution will yield the lowest production expenses by balancing (a) the costs of failure to deliver on schedule (and the costs of delivering ahead of schedule) against (b) the added production costs resulting from a departure from that schedule which minimizes production costs.

4. *Alternative routing.* In many cases it is not possible to use the ideal machine for every operation, and an alternative routing to less efficient machines must be made for some parts.

5. *Overtime.* The desirability of overtime can only be determined by considering its cost in conjunction with the many other cost factors involved.

6. *Common parts.* Many parts are common to several products and must be scheduled with several different requirement dates and quantities in mind.

7. *Spares.* Addition of spare-parts requirements to production runs provides an element of flexibility in setting production quantities so that the best machine use is obtained.

8. *In-process inventories.* Augmenting in-process inventories may increase the efficiency of machine loading, but it also adds to the requirements for working capital, to plant congestion, and to storage space requirements.

9. *Smoothing of production.* Over the longer run, costs will be reduced by using inventories to absorb a part of the fluctuations in demand, and thereby to provide a relatively stable level of employment and production. The added costs of changes in production levels must be balanced against added inventory costs, overtime costs, and costs of late deliveries.

10. *Late deliveries.* In some cases assemblies are held up by late deliveries of one or more parts, and production schedules for related parts must be modified during production to permit the assembly of lower priority orders ahead of the stalled high-priority order.

All of these factors, and probably others also, should in theory be combined into a single mathematical model which would, for any set of orders, delivery dates, inventory levels, production levels, and machine capacities, provide the best plan for production—including scheduling, routing, and sequencing for each part and each machine. But the theoretical mathematical solution is yet to be fully worked out. In any case, it would probably be beyond the capabilities of even the largest computers. Therefore, the practical solution is to use techniques of approximation or to optimize parts of the problem separately.

In general, it is possible today:

1. To find for a single product, or product group, the optimum schedule of production—including the weekly or monthly changes in production levels, work force, overtime, and inventory levels—which will meet forecast changes in sales levels. Such a schedule will provide an over-all optimum over many successive time periods.

2. To find the optimum assignment of orders to machines where there are many different machines and a large number of products, many of which can be made on several different machines.

3. To determine economic lot sizes, but only if there are no priorities or completion dates and if available machine time is unlimited.

4. To determine the sequencing of parts to machines that will optimize machine loading, but without consideration of alternative routing, priorities, or required delivery schedules.

Thus there is no complete solution available for problems of even moderate complexity. In specific cases, very significant improvements have been possible by the use of mathematical tools. In other cases, it has not been possible to find by mathematical analysis solutions which are significantly better than those already achieved by experienced production foremen and clerks with a "feel" for the situation.

If a production-scheduling process is at all complicated, and if large sums of money are involved, it would be worth a preliminary study at least, to see whether full or partial solutions might be possible which would significantly improve performance. But there are, as yet, no standard solutions. Each must be tailored to the problem at hand; consequently, an experienced and imaginative mathematical analyst is required. An experienced production man is also required to work continuously with the analyst.

B. *Techniques that have been tried:* There are several tools which have proved to be of value, but the experience in each is, so far, limited to a very few applications.

1. *Simulation.* By simulating a complex machine-scheduling and loading problem in a computer, it is possible to examine a large number of different production plans and schedules, as well as various rules of scheduling, and, by a planned trial-and-error process, to find those solutions which are the best of the many alternatives examined.

A major manufacturer has successfully used a simulation model to improve the scheduling of production and assembly shops. The model has led to the development of new scheduling rules by means of which in-process inventories have been cut in half without increasing production costs. The company also developed a management game as a means of demonstrating to plant management the superiority of the new scheduling rules over existing practices. In this way, an acceptance was obtained which would have been almost impossible to obtain by lectures and instructions.

2. *Queuing theory.* Some routing and sequencing problems are being treated as problems of waiting lines of parts accumulated behind each machine; an optimum solution can be found in some cases and approximated in others.

3. *Quadratic programing.* In adjusting production to fluctuations in sales over the longer run, quadratic programing has been successfully used to find the optimum combination of changes in production levels, size of work force, amount of overtime, and inventory levels.

4. *Linear programing.* This technique is of proven value in assigning products to machines, and in assigning orders among geographically separated plants, especially where long runs or continuous processes are involved.

5. *Mathematical logic.* When a series of operations on different parts must be sequenced across common machines, mathematical logic can be used to develop a feasible schedule.

6. *Servo-mechanism theory.** This theory is useful in smoothing

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* A theory of continuous correction of operations through measuring the variation of operations from a changing objective such as level of sales.

production by screening out random fluctuations in sales while automatically adjusting production to longer-range trends.

In summary, analytic tools can be used effectively on many production problems. Limited problems can be solved by one of the specific mathematical techniques in (2) through (6) above. Simulation provides the only feasible approach to the more general problems in which many of the factors listed on pages 90 to 92 are simultaneously involved.

Systems Reliabiilty

With the increasing size and complexity of both equipment and organizations, the problems of reliability increase in importance. On one hand, the costs of providing for reliability increase very rapidly with increasing complexity. On the other hand, the costs of equipment failure, including the cost of down time, rise equally fast.

The importance of the reliability problem can probably be best illustrated by the rapid decline in the probable service before failure of equipment as complexity increases, particularly if the components are subject to random-type failure. For example, an airborne radar consisting of 100 vacuum tubes, each rated at 1,400 hours, and 900 other components, each rated at 5,000 hours, has only a 60 per cent chance of surviving a two-hour mission. Reliability can thus become a significant limiting factor in the design of complex equipment. In general, the cost of obtaining a given level of reliability in a complex system is believed to increase roughly with the square of the number of components.

Five Ways of Increasing Reliability

In general, there are five ways in which the reliability problem can be met:

1. Increasing the reliability of individual components
2. Building in extra components in parallel
3. Increasing preventive maintenance, including periodic replacement of parts prior to failure
4. Increasing repair facilities so that down time is reduced
5. Separating and reducing in size individual pieces of production equipment and providing in-process inventories between each stage.

Obviously, not all of these methods are suitable for every type of system. Furthermore, they are all expensive. For example, increasing the reliability of individual components requires more expensive parts, more thorough inspection of parts, or, in some cases, operation at levels less than the rated capacity of the components. To take another example, the use of extra components in parallel not only increases cost and weight, but has other limitations besides. Additional components may cause failure by operating when they should not; therefore, increasing the number of components may tend to increase one type of failure while reducing the other.

In general the means of increasing reliability are justified only to the extent that they are offset by savings resulting from increased reliability. In most cases, it should be possible to make a fairly precise analytical estimate of the optimum combination of the different means of providing reliability, as well as of the optimum degree of reliability.

Prevention vs. Repair After Failure

Maintenance can be achieved either by prevention or by repair. In determining the most economical maintenance program for equipment, the costs of failure are balanced against the costs of preventive maintenance. This involves determining the probability of failure for each critical part and combining the individual probabilities into an over-all probability function for the equipment under study. The costs of preventive maintenance include the costs of maintenance labor, the costs of built-in inspection and warning devices, and the added cost of replacing parts before they have failed. The costs of actual failure include machine down time, costs of delay of other processes, customer dissatisfaction and customer loss, increased scrap, idle labor, and costs of maintenance personnel and supplies.

A key to the solution of problems of this type is the determination of the "mortality" curve for each part. Some types of equipment show a higher rate of failure during the initial period of life, followed by a reduced rate for those that survive the initial period. Others show low initial rates with high rates as wear-out takes place. Still others exhibit a constant rate of failure.

In some cases, where there is found to be a particularly high rate of failure in the early part of the life span, it is more economical to test-run all items for the initial period. Those parts that survive will thus have a greater average life expectancy than would the larger group of new parts, even though those remaining are all "used parts."

Under some conditions, preventive replacement is never economical because the value of still-functioning elements that are discarded is never compensated for by savings resulting from the reduced number of failures. Under other conditions, it will be more economical to replace a given group of elements, such as radio tubes, at fixed intervals, even though failure has not occurred.

Under the latter circumstances, the chance of a costly failure occurring between scheduled preventive replacements is considerably reduced. By mathematical means it is possible to specify the type of replacement policy that provides the optimum balance between the cost of failure and the cost of replacements.

In a similar manner, it is possible to specify an optimum periodic inspection program, or to specify the extent to which automatic checking devices should be built into complex equipment. It is also possible to strike an optimum balance between the added costs of making equipment less likely to fail and the alternative costs of preventive replacement before failure. The desirability of maintaining standby equipment in anticipation of failure can also be calculated.

While there is no ready-made set of rules or formulas for determining the optimum balance among the various means of increasing reliability on one hand, and reducing the costs of failure on the other hand, analytical tools now exist which will permit an accurate solution of many practical problems. Further, these tools have at least limited applicability to systems involving human components as well as equipment.

There is little doubt that this class of problems will increase tremendously over the next few years as we move into the age of automation. Already the "reliability barrier" is being talked about as the next major scientific challenge following the "sound barrier" and the "thermal barrier." Unlike other barriers, the reliability barrier will be of nearly as great significance to management as it will be to the designers of intercontinental missiles. Mathematical analysis will be of increasing value in answering "How much reliability is justified?" and "How can a given degree of reliability be achieved at minimum cost?"

Maintenance

Many of the analytical techniques applicable to maintenance are discussed under the heading of "Systems Reliability." Others apply specifically to maintenance problems.

Queuing Theory in Sequencing Repairs

Determining the sequence of repair jobs is one maintenance problem in which mathematical analysis is useful. Both the estimated time required for repair and the relative importance of each job must be considered. The setting of repair priorities in turn influences the optimum size of repair facilities. For if the priorities are improperly set, important repair jobs will be delayed longer than necessary, and larger than necessary repair facilities will appear to be required. Queuing theory provides the theoretical framework for the solution of problems of this class.

Probability in Planning Parts Stocks

The determination of the optimum levels and mix of spare-parts stocks is often highly complicated. However, probability theory can be used in making such a determination. Given the expected mortality tables for each part, it is possible to balance the costs of production and storage against the costs of failing to provide enough parts. Similarly, if a fixed amount of money is available for spares, it is possible to calculate the optimum mix of parts that can be purchased for that sum. Both the cost of each part and its probable rate of use will enter into the calculation.

To make the point with an extreme example, assume that there are ten parts. During a given period, each of nine parts has a 10 per cent chance of being needed and a tenth part has a 20 per cent chance of being needed. The cost of the tenth part, however, is equal to the entire fund available for parts purchase, while one of each of the nine other parts could be purchased with the money available. Thus, even though the probability that the tenth part will be required is double the probability that any one of the other parts will be needed, the purchase of the nine parts will be preferable to the purchase of the tenth. For if the tenth part only is purchased, the probability of a breakdown for which there is no spare part is 82 per cent, while if the same money is spent on one each of the nine parts, there is only an 18 per cent chance of breakdown.*

Often, total weight or total volume must be considered in calculating the optimum mix of spares. Weight is a factor when, for example, spare-parts kits are to be assembled for air delivery; and total volume is often governed by warehouse limitations. In either case, the method discussed above for calculating optimum mix from the cost standpoint can be used.

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* For the sake of simplicity, the possibility of a second breakdown for a given part is ignored in the example.

Techniques for Locating Warehouses

Other problems may involve complicated decisions on the location of spare-parts depots. It may be possible to carry spares centrally in a single warehouse, to maintain stocks at regional warehouses, to locate all spares with the equipment for which they are intended, or to combine two or all of these arrangements. If spares are maintained centrally, the size of the stock will be smaller than would otherwise be necessary; but the costs of delivery, possibly by air, and the costs of delay will be greater. If stocks are decentralized, the size of the stocks will have to be much larger, although the costs of delivery and of delay will be less. Linear programing, quadratic programing, and probability theory provide tools for the solution of problems of this type. Very complex problems probably can be solved only by use of simulation models.

United Airlines has developed a major simulation model covering the operations of maintenance and service facilities at an air terminal. Warren Alberts, Director of Industrial Engineering, writes:

> The project involved a basic study of the whole process of providing serviceable aircraft to meet schedules over our 14,000 mile system. Our first objective was to determine and analyze the true nature of that operation and measure the variables which existed. The second was to develop a means or a model which could be used to test the effect of changes in policy, maintenance concept, schedules, facilities, and manpower. . . . The station model as it was set up on the 704 computer, will enable us to simulate 100 days' actual operation—say, at a station like Newark—in a matter of minutes. By varying the input of such items as manpower, facilities, schedule, or maintenance policies, we hope to compare outputs and get an indication as to the best course of action. As output, the model will give us idle manpower, utilization of maintenance docks, schedule delays, maintenance working and waiting time, and so on.*

Purchasing

Several applications of mathematical analysis have been made to specific problems in the purchasing field.

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* Warren E. Alberts, "Operations Research in Aircraft Maintenance," *Operations Research, A Basic Approach,* Special Report 17, American Management Association, 1956.

Linear programing has been used by a mid-west steel tube mill to determine what products to make, and what to buy from other mills for resale through the company's sales organization. The analytical solution showed that profits could be increased substantially by outside purchase of certain products, even though the mill had sufficient productive capacity to make them.

Similarly, a large purchaser of parts has used linear programing to analyze a large number of bids. The bids presented numerous price and quantity alternatives, both on individual parts and on groups of parts. Some bidders, who submitted several bids, had limited equipment and could accept only certain combinations of orders. Under these circumstances, selecting the combination of bids that would minimize total costs presented the purchaser with a formidable problem. Without linear programing, he would have been forced to use tedious cut-and-try methods of approximation. In general the use of linear programing probably is not justified on a "one-shot" basis, unless the value of the items being purchased is quite large and the complexity of the analysis beyond the capability of simpler approaches. However, if a manufacturer is regularly in the market for a large number of parts, he may find it worth while to set up a linear programing form of analysis on a computer that can be used repeatedly to analyze bids.

Another use of mathematical analysis in purchasing is in weighing the advantages of a low bid with a high probability of late delivery against the advantages of a higher bid with a better chance of meeting a promised delivery schedule. E. J. Isaac of the Stanford Research Institute, has formulated a solution by which linear programing is used to determine how low a probability of on-time delivery can be tolerated before it becomes better to accept a higher bid with a high probability of delivery.*

Probability theory has been used to determine the number of spare parts that should be ordered with a new machine. On the one hand, extra spare parts may never be used; on the other, an inadequate order may result in a critical part's becoming exhausted before the machine is junked. If adequate estimates of the probability of part failure and of the life of the machine can be made from past records, it is possible to compute the number of spares which will minimize the total of the costs of spares and the probable costs of running short. The factors to be taken into consideration include: the initial cost of the part, the costs of storage and of stores maintenance, the probability of need for the part, the costs of shutting down a machine or a whole line for lack of a spare, the costs of a special production order for additional spares, and the length of time required for their production.

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* E. J. Isaac, "Note on Selection of Capital Equipment with Uncertain Delivery Date," *Operations Research,* Vol. 4, No. 3, June 1956.

Transportation

The applications of advanced analytical tools to problems of transportation fall largely in routing and scheduling.

The Stanford Research Institute, working with the Southern Pacific Railroad, has developed a system by which the various types and grades of empty freight cars can be dispatched to pick up their next loads in a way that minimizes the over-all costs of movement, including both in-transit time and distance traveled. Since a major railroad may have as many as 10,000 empty cars on hand at any one time, the movement to reloading points can be a major cost. Further, the more efficiently the car pool can be used, the fewer total cars will be required by the railway. This problem has been solved by use of mathematical techniques similar to linear programing. The complexity of the calculations required is such that only by use of an electronic computer is the process possible. Data collection, as in most important problems, has been a major obstacle. It was found necessary, as a part of the program, to develop an entirely new system of reporting on empty-car status.

Several studies on automobile traffic flow have been made by the Port Authority of New York and other public agencies. One study, for example, has determined the optimum conditions of speed and spacing in order to move the maximum number of vehicles per hour through New York tunnels. It is interesting that, as speed increases up to a critical value, volume also increases; and as peak speed goes higher, volume again contracts because of alternate bunching and thinning of cars.

The techniques of linear programing are also of considerable value in the solution of some transportation routing problems. Others, where it is necessary to optimize the routing and scheduling through several successive trips for each ship or aircraft, will require the more difficult techniques of dynamic programing. Most of the work done so far has been for the military services. Commercial systems, however, are considerably more complicated than military systems, both because of competition among carriers and because of the complex nature of the demand of customers for specific times of arrival or departure. Therefore simulation may prove to be the only feasible means of handling the complexity of many transportation problems.

Research and Development

The principal application of advanced analytical techniques to research and development has been in helping to set more precise requirements for applied research and for development in industrial laboratories.

Weapons Systems Analysis by Defense Contractors

Most of the major defense manufacturers maintain large operations research staffs whose function is to study potential weapons systems ten to twenty years ahead. The analysis of alternative weapons systems, and the probable enemy systems they will face, brings out the fundamental elements of future combat and the characteristics an aircraft or missile must have if it is to operate successfully in such environment. These characteristics, in turn, set the operational requirements and limitations of speed, weight, climb, ceiling, fire power, and electronic guidance to which the designers must adhere if their companies are to obtain developmental contracts and, later, production orders.

Simulation of Railroads

Similarly, the Westinghouse Airbrake Company has spent a great deal of effort and money in simulating the operation of railroads in order to guide its research and development staffs in the design of new equipment systems for the control of railroad traffic. The first element simulated was the railroad classification yard. By studying the operation of the model, it was possible to find the requirements for the optimum operation of an actual yard of any given capacity. Since one of the critical factors in yard efficiency is train size, Westinghouse next developed models of line operations and, from this, was able to set conditions for the over-all optimization of line operation and yard operation functioning as a complete system. These studies showed that new automatic equipment would be necessary to operate rail transportation systems at optimum levels. They also provided the performance characteristics for the design of such equipment and estimates of the savings in operating costs that could be realized by its use.

Other Techniques

Search theory, another tool of analysis which has been developed by military operations research, may also be applicable to industrial research programs. Search theory provides a theoretical means for allocating search efforts for hidden objects under conditions in which one knows only (a) the relative probability that the object sought will be found in one of several areas and (b) the operating characteristics of the tools of search. The mathematical model gives the relative search effort that should be expended in each area. George Kimball of the Massachusetts Institute of Technology, in writing on industrial applications of military operations research, has commented, "Many industrial-research programs are in essence search plans, and it could very well be that search theory could be applied to the design of such programs."

Some of the companies engaged in the development and marketing of new chemicals and drugs are beginning to use mathematical analysis to help their researchers in identifying, from among tens of thousands of possible organic compounds, those which have precisely the required combination of physical and chemical properties as well as the absence of undesirable side effects.

In summary, advanced tools of mathematical analysis have demonstrated their value, in a limited number of cases, in setting performance characteristics for applied research and development programs. These tools may also be valuable in identifying the lines of search that are most likely to yield results in a specific research program.

MARKETING

Distribution and Sales

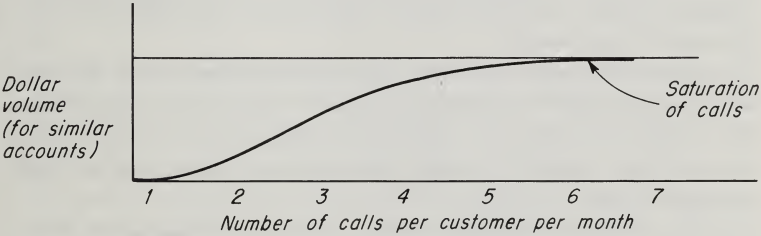
Mathematical analysis has been applied to several industrial problems of distribution and sales. A representative group of applications —some tested and some potential—is listed briefly below.

Examples of Problems

A. *Effect of promotions*: A study made by John F. Magee of Arthur D. Little, Inc., isolated the effect of promotional activity on the sales of individual dealers. As a result of being able to predict the probable increase in sales resulting from promotion, it was possible to identify those groups of dealers for whom promotion would show the greatest return. It was also possible to estimate that level of promotional activity which would maximize the profit return.*

B. *Frequency of salesmen's calls and dollar volume of sales:* The original objective of a study made by Russell Ackoff of Case Institute* for the Lamp Division of General Electric was to determine the optimum number of salesmen in relation to the market so that expansion of the sales force over the years ahead would be properly planned. The analysis of the relation between frequency of calls and dollar sales volume for each customer, contrary to expectations, showed no correlation. In an effort to find some significant relationship, a variety of other tests were tried, including a breakdown of customers into groups and a breakdown according to whether the number of calls to a customer had been increasing or decreasing during the last two years. None of the tests showed any positive relationship. It was finally concluded that the current frequency of salesmen's calls was well over the threshold of saturation so that increasing or decreasing the number of calls made little change on volume of sales.

A sales-response curve might look something like this:



As the number of calls increases, sales will increase less and less until the flat part of the curve is reached where no further increase can be expected. It was the conclusion that the Lamp Division had been operating well into this area; hence changes in number of calls had no effect on volume. As a result, a program of carefully reducing the frequency of calls was undertaken, with the objective of finding the point at which volume would begin to drop off. Once the nature of the curve in this area becomes known, it will be possible to estimate the optimum number of salesmen and to allocate their effort in the most advantageous manner.

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* J. F. Magee, "The Effect of Promotional Activity on Sales," *Operations Research for Management*, edited by McClosky and Trefathen, Johns Hopkins Press, 1954.

C. *Salesmen's quotas and compensation*: The establishment of sales quotas and sales compensation plans which take into consideration simultaneously the characteristics of each territory, the most productive classes of accounts, the over-all effects of levels of economic activity, and the profit margins on individual products, is in some cases possible with today's analytical tools. Linear programing may be of value in complicated situations involving many different products and market characteristics.

D. *Relative profitability:* Linear programing and other techniques offer the possibility of evaluating, on a continuing basis, the relative profitability of individual products so that sales effort can be concentrated on these items.

E. *Financial help to distributors:* Many distribution systems are plagued with the inability of their accounts to finance, on a cash basis, large enough orders to permit economical distribution. Gasoline service stations are frequently unable to take a full truckload of gasoline, even though their own tank capacity is adequate. Under these circumstances, analytical techniques may help answer the question of what is the optimum combination of credit expansion, price concession on large orders, and added costs to the distributor for delivery of small orders.

F. *Evaluation of distributors:* Many manufacturers may be using distributors whose contribution to profit is marginal or even negative. Analysis can show both the lower cutoff point for such distributors and the avenues by which marginal distributors can be made more profitable.

G. *Retail Sales:* Edward H. Smith, a scientific consultant in Washington, was asked to develop a means of reducing pilferage in a large department store. In carrying out the assignment, he tagged several thousand dresses over a period of weeks and followed their course from the receiving department to the sales floor and finally to the customer. An unexpected but more valuable product resulted from the analysis of the collected records by a trained statistical mathematician. It was found that in every case a given percentage of each lot would never sell and that the number sold each week would be proportional to the balance of the salable dresses which were still unsold at the beginning of the week. This could be expressed in simple mathematical terms and as a result, the sales from each lot could be accurately forecast for each succeeding week. The individual dresses which would not sell could not, however, be identified in advance. Hence it was impossible to segregate the lots. The schedule of sales might, for example look like this:

Sales from a 1,000-Dress Lot

Week	*Number sold*	*Balance*
1	280	720
2	168	552
3	101	451
4	60	391
5	37	354
6	21	333
7	13	320
8	8	312
9	5	307
10	3	304

By use of this type of information, it is possible to calculate the number of weeks the unsold balance of a lot should remain on the floor before it is marked down and the number of weeks before the remainder should be removed from the floor and, if necessary, given away.

In the above example, 300 dresses could be expected never to sell at full price. At some point, possibly the end of the sixth week, the loss incurred by marking down 33 remaining dresses which ultimately could be sold at full price, together with the 300 that would never sell, would be less than the loss suffered by keeping the unsold balance of the lot on the floor and thus not being able to use the space for a new lot which would initially sell much more rapidly. When the specific initial prices, mark-downs, profit margins, and costs of floor space are known, the problem can be expressed mathematically and solved to find the length of time each lot should remain on the floor before mark-down or before being disposed of by other means.

Total Marketing Expenditures

Determination of the optimum level of marketing expenditures under competitive conditions has always been a difficult problem. If a competitor increases his share of the market by raising his expenditures on sales, sales promotion, advertising, and product differentiation and improvement, how far should one go in trying to meet this competition by raising one's own level of marketing expenditures? Clearly, there is some point at which the added expenditures will bring back a lesser amount of added profits.

Robert S. Weinberg of IBM has developed a technique of analysis which has been applied in a small number of situations to problems of this type.* The objective of such analysis is to project the costs of securing a greater share of the market in the fact of competitors' actions and to equate these costs to the added profits they will bring. Further, the analysis gives the company an opportunity to compare its costs of holding or expanding its share of the market with the corresponding expenditures of its competitors.

A. *The cost of increasing share of market:* As a first step in the analysis, company marketing expenditures for several years past are collected. They include all expenditures which are intended to maintain or increase sales volume, such as sales costs, advertising, promotion, product development, and market research.

Comparable data are collected on competitor operations. Obviously this must be an estimating operation: but published financial statements, trade reports, and other "intelligence" can usually provide adequate data. A next step is to relate past shifts in share of the market to the individual marketing expenditures per dollar of sales for each competitor during the period in which the shifts have occurred. By comparing the company's marketing expenses per dollar of sales with those of competitors, a marketing expenditure ratio can be obtained. This ratio, in turn, when plotted against the net change in the company's share of the market which has occurred in the past, provides a means of estimating the added expenditures necessary to increase the company's share of the market by any given percentage. This relationship can be plotted as shown in Figure 7. As is shown on the chart, the cost of increasing the share of the market goes up more rapidly than does the share of the market itself. This chart might be used as follows:

> Suppose the company management decides to increase its share of the market in 1958 to 20 per cent, an increase of 2 percentage points over 1957. Using Figure 7, or its equation, the ratio of company expenditures to competitors' expenditures per dollar of sales, which

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* R. S. Weinberg, "Multiple Factor Break-even Analysis," *Operations Research,* April 1956, pp. 152-186.

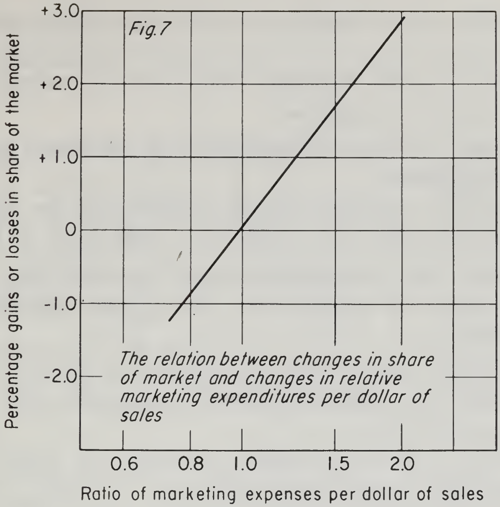


Fig. 7

The relation between changes in share of market and changes in relative marketing expenditures per dollar of sales

is necessary for such an increase in a single year, is found to be 160 per cent. This means that to achieve this expansion, the company must spend, for each dollar of sales, 1.6 times its competitors' expenditures per dollar of sales. It is next esitmated that competitors will spend $22 million in 1958 on marketing. The total industry market is estimated at $490 million. On this basis the company would have to spend $9 million on marketing to capture 20 per cent of the market, while the cost of holding its present 18 per cent would require a marketing ratio of only 98 per cent, or a cost of $4.7 million.

The increase in dollar sales in going from 18 per cent to 20 per cent would be only $9.8 million. If the direct manufacturing costs of this added volume were $5.6 million, the proposed expansion would be unprofitable in 1958, as the company would have spent an additional $9.9 million to increase sales revenues by $9.8 million. However, once a larger share of the market has been captured, its retention becomes less costly; therefore calculation of the optimum share of the market should also include incremental returns over a period of years. On this basis the expansion of share of the market might prove to be profitable.

A further use of this type of analysis is to show the increased earnings that will accrue from an improvement in the efficiency of the marketing dollar. In other words, the value to the company of lowering its costs of sales in relation to its competitors' costs can clearly be demonstrated. The analysis can also be used to measure the profitability to a competitor of obtaining a larger share of the market.

B. *Limitations of this technique:* This form of analysis is subject to certain important limitations:

1. There may be considerable difficulty in measuring competitors' marketing expenditures.

2. The analysis assumes that all expenditures are equally well spent. It does not take into consideration differences in the quality of advertising, sales management, promotion, and style. Therefore it will not work well in situations where these qualitative factors are predominant.

3. It is at very best a rough approximation; and until the technique is considerably refined, it should be regarded only as a guide and not as a proven rule.

Nevertheless, it has been useful in practice in finding the best level of marketing effort under conditions of strong competition, especially where marketing costs are high and style is not a major factor.

In general, it appears that, for certain types of marketing problems, solutions, or at least approximate solutions, can be found by analytical means. But all such applications should be approached in the same way one would approach a research and development program. There is no certainty of success; the problem may change many times during the period of study; and the analysis may cost more than was anticipated. The best way to increase the chances of pay-off and to hold down costs is to use the most skillful people available to direct the work. In summary, the types of marketing problems which may in some cases be solved by analytic means include:

1. Allocation of sales effort
2. Setting of sales incentives and compensation
3. Evaluation of promotional devices
4. Use of distributors
5. Distribution financing
6. Budgeting of marketing expenditures.

Pricing and Bidding

Pricing is one of the marketing problems in which few applications of mathematical analysis have yet been made. The importance of pricing to the success of any business, however, warrants consideration of the new techniques as possible aids to judgment in this field. The analytical work that has been done in the field of bidding is encouraging and has already had a few useful applications.

Pricing

Analytical work on price determination usually founders on the rock of price-demand relations. If one were able to make fairly accurate estimates of the relation between price and sales volume, the optimum price policy could, in many cases, be determined by rigorous analytic means. Economists and mathematicians writing on the subject often have jumped over this obstacle by assuming a "price-demand" curve, and then going on to show how to find the optimum price with this curve. A man with an excellent reputation and on the faculty of an outstanding university has written an article in a professional journal laying out the mathematical means of determining the optimum price under a variety of conditions. The paper starts off with the statement, "The merchant is assumed to know the number of units that will be demanded at each given price. . . ." But this is seldom true. Until analytical means are developed for gaining this knowledge, pricing will remain largely an intuitive art practiced by the experienced and skillful merchandiser.

This is not to suggest that price-demand curves are beyond the capabilities of analytical solution. Rather, the analytical approach will require breaking new ground and may be a costly and time-consuming endeavor. Because both prices and demand are always affected by many rational and irrational forces in the economy, it will be a major job to isolate the effect of each factor so that the direct impact of price on volume can be measured. This will require a willingness to conduct experiments in pricing and to collect large amounts of data. The technique of factorial analysis may prove to be particularly useful, both in experimentation and in the analysis of the resulting data.

Insofar as costing can be made more accurate by analytical means, pricing will, of course, be facilitated in those areas in which prices are negotiated directly between the supplier and the purchaser before production begins. Analytical tools of costing, such as linear programing, may from time to time be of value in presenting rate cases before government regulatory bodies.

Bidding

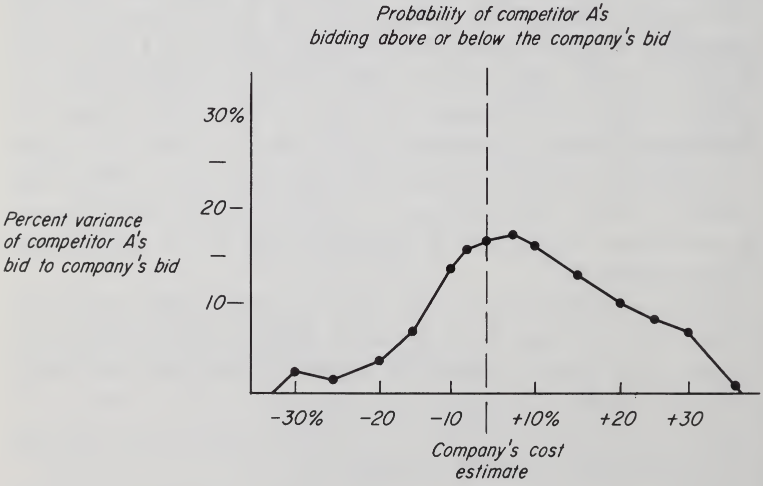
Work done by Lawrence Friedman of Case Institute on competitive bidding strategy is now being tested by at least one large corporation.* It provides a method for estimating the optimum bid to make when bids are to be sealed. The two principal factors entering into the determination of the optimum bid are:

1. The probability of success with bids at alternative prices
2. The expected profits from each alternative bid.

As the maker of automobile bumpers, for example, increases his bid, his expected profit increases, but the chances of his being low bidder decreases. (If the bidding were for the purchase of securities or for oil leases, for example, the high bidder would, of course, have the greater chance of success but a lower profit expectation.)

The Friedman technique identifies the bid that maximizes probable profits. In this technique, the probability of winning is determined by analyzing the past behavior of competitors.

A. *The probability of winning:* A first requirement is the knowledge of past bids submitted by each competitor in similar competitions. From this information, one can develop, for each competitor, a probability curve showing various percentage points by which his bid can be expected to vary from one's own bid (or estimate of cost). For example:



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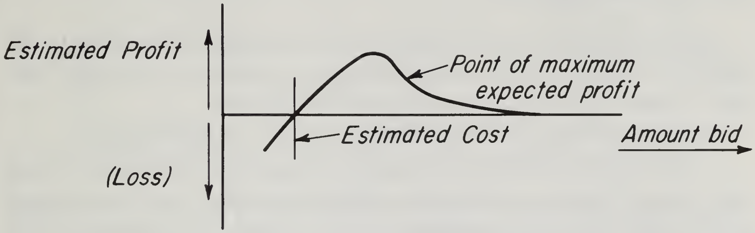
* Lawrence Friedman, "A Competitive Bidding Strategy," *Operations Research,* Vol. 4, No. 1, p. 104. February 1956.

Thus, the probability that the competitor will bid above the company's bid will be equal to the shaded area under the curve. Similar curves can be estimated for other competitors, and these can then be combined into a single probability curve for all competitors.

B. *The probability of profit:* Next, the probability of variances between the company's past bids and the actual cost can be estimated from past records of successful bids. This information, together with the probability of winning on a competition with any given bid, enables one to estimate the probable profit for each alternative bid, as shown below.



Obviously, this tool should not be used blindly, but only as an aid to judgment. There may be other factors which dictate a different bid. For example, if the bidder has a full production schedule, he may decide he is only interested in another contract if he can make a return of at least "X." Under these circumstances, he selects his bid price, refers to the distribution of past bids of competitors, and decides whether the probability of success is sufficient to warrant his entering a bid. Equally, if the bidder is primarily interested in maintaining employment during a slack period, he may reduce his bid below the optimum in order to increase his chances of being successful, even at the cost of profits. Further, the bidder may have additional information about the probable bidding of competitors. Such information can be applied either by altering the bidding curves of competitors or by making a judgmental correction from the indicated optimum bid. In either case the analysis is an aid to judgment, not a substitute for it.

Advertising

Advertising may offer a fruitful area for the application of analytical techniques. Like pricing, however, it is an area of so many difficult-to-measure factors that considerable work will need to be done before mathematical analysis can be applied to any great extent.

Difficulty of Isolating Effect of Advertising

The essential problem, of course, lies in the difficulty of relating advertising directly to sales. While it is possible to determine what advertising is read by which people, and how much they remember, it is very difficult to isolate the effect of advertising on the consumer's decision. Consequently, advertising budgets are usually set as a given percentage of sales, even though there is seldom much objective evidence to support the resulting figure or to indicate which combination of media will yield the greatest return.

An Analytical Approach

Arthur D. Little, Inc., has developed an analytical approach to the planning of advertising which appears to provide a practical tool, and a basis for further research and development.

In essence, there are three elements of the Little analysis:

A. *Rate of sales decline when advertising stops:* It has been found that sales of certain products for which advertising has been stopped will fall off at a rate proportional to the volume of remaining sales. For example, sales of a given product may have been found by experiment to drop off at a rate of 50 per cent a year. If, at the time advertising is stopped, sales are at the rate of 100,000 a year, then: at the end of the first year, sales will drop to a rate of 50,000 a year; at the end of the second year, they will be at a rate of 25,000 a year; and at the end of the third, 12,500 a year. The rate of decline will, of course, vary widely from product to product; but for individual cases, the specific rates have been estimated by controlled experiments in which advertising is temporarily cut off in a given area.

B. *Relation between advertising effectiveness and share of market:* It has also been found for certain products than any given level of advertising will increase sales along a curve which tends to flatten as the share of the market increases. At a definable point the curve will reach a saturation level, beyond which advertising will have little or no effect. This curve fits the general rule that the increase in sales per dollar of advertising will be proportional to the difference between the actual level of sales and the saturation level.

C. *Effectiveness of advertising per dollar spent:* For each product, this will be a changing value, depending upon the advertiser's specific share of the market; but it has been possible to define this curve mathematically so that, for a given product, it can be completely plotted from a limited number of measurements. Figure 8 shows a

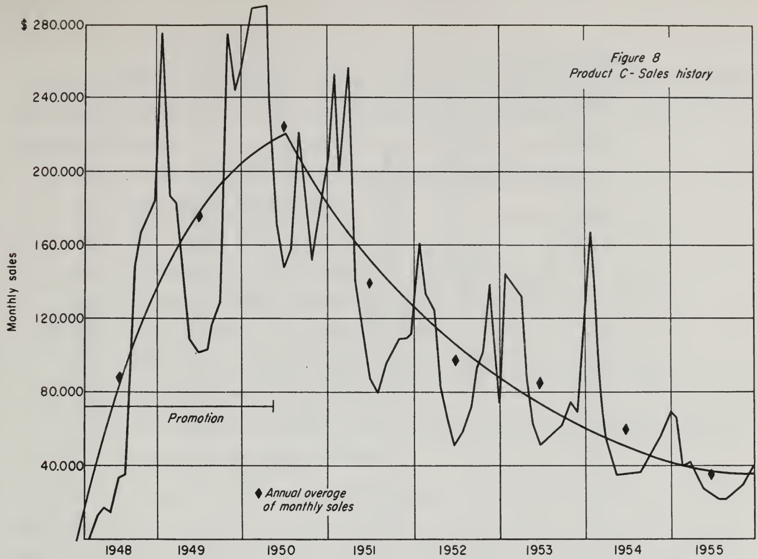


Figure 8
Product C - Sales history

Source: M.L. Vidale and H.B. Wolfe, Measuring Advertising Effectiveness, Arthur D. Little Inc. Cambridge, Mass.

typical curve of buildup and decay. Figure 9 shows how the decline of an unadvertised product is temporarily halted by two short bursts of advertising.

All three of these factors are combined into a single mathematical expression by means of which it has been possible to estimate the following:

1. The increase in sales that can be expected from a given advertising campaign
2. The most efficient level of advertising expenditures during the buildup of a new product
3. The optimum steady-state advertising budget for an established product
4. The best length of an advertising campaign designed to boost sales
5. The best allocation of a given advertising budget among different products
6. The optimum advertising effort to provide a given "return on investment"

In its present form, there appear to be two major limitations to this technique. First, it does not consider the effects of advertising and marketing programs of competitors. Second, it does not consider other means of increasing sales, such as promotion, price concessions, direct sales effort, style, and product differentiation. This approach

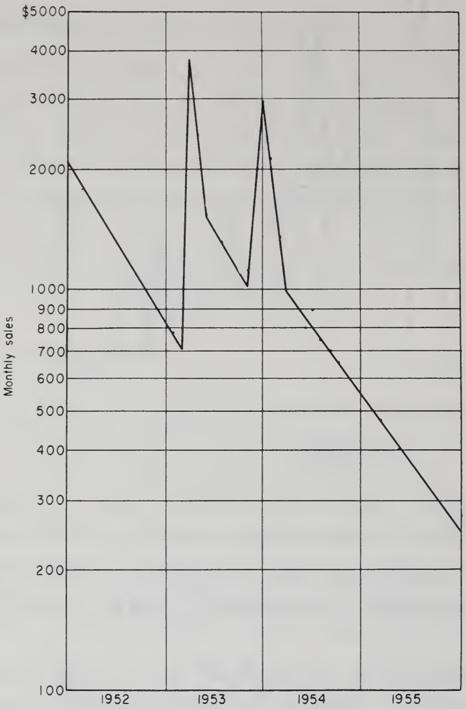


Figure 9

Product C - sales history

Source: M. L. Vidale and H. B. Wolfe, An Operations Research Study of Sales Response to Advertising, Arthur D. Little, Inc., Cambridge, Mass.

can, however, be incorporated into broader analyses that do consider these other factors.

In summary, a principal difficulty in conducting research in marketing is the difficulty in separating out the many factors which are acting simultaneously to influence the purchasers' decisions. The powerful tools of statistical analysis, especially factorial analysis, provide encouraging possibilities for further work which may ultimately be of considerable value in marketing decisions. The possibilities of significant advances in the solution of basic advertising problems are sufficiently attractive to justify analytical research being undertaken.

Inventory Control

In many industries the control of inventory is intimately related to the scheduling of production, and the two should be treated as a single problem. But where inventories are maintained by purchases, the optimum inventory level is that which minimizes the total of two classes of costs:

1. The costs associated with ordering and storing:
    - (a) The costs of storage
    - (b) The costs of capital
    - (c) The costs of ordering and handling
    - (d) The cost differentials for orders of different sizes
2. The costs of running out of stock:
    - (a) The costs of special services, such as air delivery
    - (b) The direct costs of lost sales
    - (c) The indirect costs of lost sales, such as permanent loss of accounts to competitors

Some of these costs are relatively easy to compute; others are more difficult. The costs of lost sales usually can only be estimated roughly. Yet inventory policies that ignore these costs will often be badly distorted and needlessly expensive in terms of lost sales.

Runout versus Storage Costs

In many cases, inventory withdrawals have a large unpredictable or random element. In this situation, the costs of running out of stock must be expressed in terms of probability distribution rather than fixed costs. In such cases the probability for each of several levels of sales is calculated. For example, the probability distribution of sales in a given month might be as follows:

Number of items sold in month	*Probability*
1	12%
2	17
3	22
4	20
5	16
6	10
7	2
8	1

If the costs of holding and storage are $15 per month per unit, and if the costs of running out are $100 per lost sale, how many should be stocked as protection against runout? The probable cost of lost sales for any given size of inventory is equal to the costs of being unable to fill orders of various sizes above the inventory level, multiplied by the probabilities that such orders will actually occur.

The optimum safety stock to maintain is that which will minimize the combined costs of carrying inventories and of sales lost by runouts. It can be calculated as follows:

		Cost of sales lost through runouts			Combined probable costs of lost sales and inventory maintenance
Inventory safety stock (units)	Cost of storage per month	Number of sales lost	Probability (from table page 115)	Cost	
1	$15	1	.17	$ 17	
		2	.22	44	
		3	.20	60	
		4	.16	64	
		5	.10	50	
		6	.02	12	
		7	.01	7	
		(Total probable cost)		$254	$269
(For other values of safety inventory, the calculations are similar)					
2	30			$166	$196
3	45			95	140
4	60			46	106
5	75			17	92
6	90			4	94
7	105			1	106

In this example the optimum safety stock is thus 5 units, which will result in a combined cost of inventory holding and lost sales of $92. If, instead, the safety inventory were set at 3 units, which represents the most probable level of sales for the period, the total costs per month would be $140. On the other hand, if the inventory were set sufficiently high to preclude (theoretically) the possibility of lost sales, the total costs would be $106. (With few exceptions, there is always some small possibility of running out, no matter how high the inventory; hence it will be impossible to have complete protection from runout.)

Other Factors to Consider

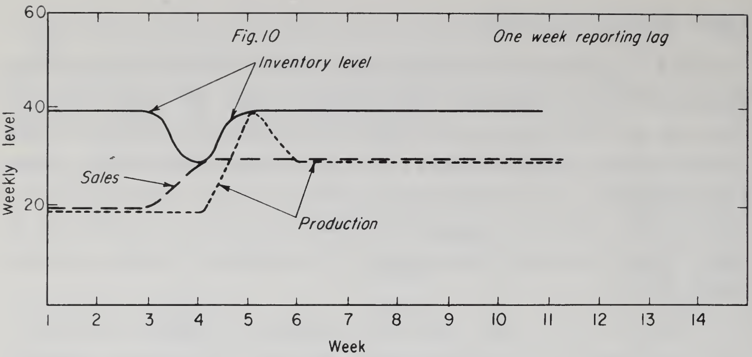
A. *The lag between ordering and delivery:* If this is very long, there may be several orders outstanding at any one time. The decision rule on reordering, under these circumstances must take into consideration the expected future deliveries against outstanding orders, the probability of variance in future sales, and the expected delivery date for the order being placed. It is also possible to include in the mathematical analysis the element of uncertainty in the promised delivery dates.

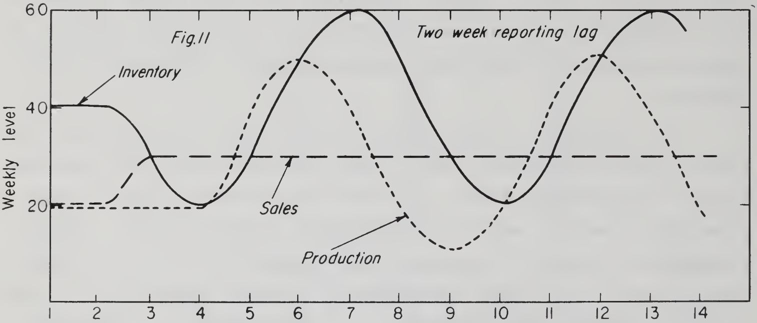
B. *Costs over the long term:* The inventory problem may also involve the question of optimizing the total costs over several successive time periods, such as weeks or months. In other words, it may be that considering all time periods together, rather than finding the optimal solution for each time period separately, will yield the better solution.

C. *The reporting system:* Delays in the reporting system can also cause serious distortions and swings in production and inventory levels. If production levels are based on inventory levels, a lag in reporting inventory stock can sometimes produce violent swings in production. Assume, for example, that production for each week is set as equal to sales plus the difference between standard and actual inventory levels. If the inventory is below standard, production is increased to make up the difference. If the inventory is above standard, production is decreased by the difference. If, as shown in Figure 10, reports on sales and on inventory levels in the warehouses take two weeks to arrive at the factory scheduling office, violent and continuing oscillations in production will be produced by a single change in level of sales. In this example, sales increase in the third week from 20 to 30 units and remain at the higher level for the following weeks. This simple change will produce a continuing oscillation in inventories between 20 and 60 units, even though sales remain constant at 30 units.

If the delay is reduced to one week, the immediate swing in production is reduced considerably and stability is reestablished quickly, as shown in Figure 11. If, on the other hand, the delay in reporting is increased to three weeks, even more violent oscillations result.

It therefore may be important to include in an inventory model expected lags in reporting of sales and inventory levels. For if these





lags are known, the model can be adjusted to prevent recurring oscillations.

Mathematical servo-mechanism theory, originally developed to control radar tracking of aircraft, and to minimize the problems of over-correction of antiaircraft aiming, can have direct usefulness in handling problems of this sort. It provides a means for filtering out the short-term and random fluctuations in sales, and thereby adjusting production to longer-term trends in a way which stabilizes production and inventory levels.

Part IV.
CONCLUSION

How These Techniques Stand Today

Some analytical tools are of proven value, others are of considerable potential value, while still others will have little practical application unless and until the theoretical work is pushed considerably beyond its present point. Linear programing, queuing theory, probability theory, Monte Carlo methods, and simulation techniques are of proven value. Dynamic programing, quadratic programing, and operational gaming have been used successfully in a few cases to solve practical problems and appear to have considerably greater potential application. Symbolic or mathematical logic, input-output analysis, and search theory have today limited application to special problems. Formal game theory appears to have little or no practical application in its present state of development.

It is my own belief that probability theory, simulation, and mathematical programing will have a major impact on management over the next few years.

Requirements for Successful Application

The successful use of these new tools by management requires both trained mathematicians and skilled management specialists. It will also depend on:

A. *The Selection of the Problem*

1. A problem must be sufficiently important to justify the effort and cost. Many firms have approached the use of mathematical analysis by selecting a small problem of peripheral value, with the idea of gaining experience before tackling a problem of major significance. Often this has resulted in failure and disillusionment because:

(a) The solution of a small problem turns out to be as difficult and costly as the solution of a problem of basic importance would have been. As a result the cost and effort are not justified by the limited results.

(b) It usually turns out that the problem has been inadequately defined, and it soon becomes apparent that the original approach must be greatly broadened. Management at this point often loses patience with the analysts for having failed to solve the problem they were set to solve, while not understanding the need to broaden the problem's definition.

(c) The analyst may be assigned to an organizational element appropriate to the original problem, but not broad enough to handle the problem as it is redefined.

The alternative is to concentrate on those problems to which solution will be of major value to the company. Then allow plenty of time for the solution to be worked out, and be patient with the inevitable false starts and delays. On the other hand, the problem cannot be so broad and diffuse that it cannot be solved. The best compromise between these two extremes may be to break a broad problem into several pieces which can be tackled one at a time.

> Operations research has been of considerable value and potentially is of far greater value, but it is not without perils of its own. For some problems, the new techniques work wonderfully; for others they do not, and it is indispensable that we cultivate the ability to discriminate between the cases where the techniques can be profitably applied and those where they cannot. This will call for intimate participation and consultation between decision makers and advisers, not just the farming-out of some particular vaguely specified problems. Sophisticated analytical tcehniques and common sense must be blended to yield fruitful results. They must complement, not substitute for, each other.*

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* Malcolm Hoag, "Operations Reserach—A New Science," *Journal of Business*, University of Chicago, July 1957.

B. *A Definition of the Problem Sufficiently Broad so that Dangerous "Suboptimizations" are Avoided*

In other words, one problem must not be solved by creating other and possibly worse problems.

C. *Adequate Data of Sufficient Accuracy, Collectable at Reasonable Cost*

The problem and its form of solution must first be carefully thought through to avoid either collecting irrelevant information or failing to provide the essential data. Often preliminary models will reveal which factors are critical in the solution of the problem and which are not. Advanced statistical techniques are of great value in the design of data collection and analysis programs to produce the required information at minimum cost. Finally, a skilled analyst will be needed to adapt the form of solution to the limitations on available data and, most important, to calculate the degree of confidence which can be placed in the results.

D. *The Selection or Development of a Technique of Solution which Provides the Proper Balance Between Simplicity and Complexity*

Any model is, by definition, an abstraction from reality. Many elements of reality, therefore, must be eliminated from the model. The essential skill in model building is to include those elements which are critical to the solution and to eliminate all which are not essential. If a few figures on the back of an old envelope suffice, so much the better. The selection of safe shortcuts is clearly an art. In the more difficult problems, only a mathematician with a depth of training and experience can develop a feeling for the method which provides sufficient accuracy without unnecessary refinements and frills.

E. *The Testing of the Solution to Uncover Errors*

Since no model can adequately represent reality, there must always be a real question of the validity of the solution. Here the analyst must be encouraged to make sensitivity or stability analyses and to check his model by trying it out on the past before it is accepted as a guide to the future.

F. *The Selection of the Proper Criteria by which to Judge the Relative Merit of Alternative Solutions*

The criteria selected for a given problem will be more specific than will the over-all criteria for the success of the company. If misleading conclusions are to be avoided, the criteria of a good solution must be known to be consistent with the over-all corporate goals and objectives.

G. *The Selection of a Problem which is Not so Urgent that a Crash Solution must be Found*

Good work takes time. However, if a model has already been built and proved out when a crisis arises, it may be of great value in finding a better solution than would otherwise be possible.

As a result of the developments of the past several years, it appears that we are now approaching a point where mathematical analysis can be of great value in the organization of very large material and human systems. We now accept as normal the very heavy costs of design for complicated machines of today. The design of a modern aircraft requires several thousand man-years of the time of scientists, engineers, and technicians. Ships, building, and power plants require similar investments. It is not improbable that we also will soon be making very large investments of technically skilled manpower in the design of analytical systems for the integrated planning and operation of major man-machine organizations, such as a world-wide complex for the production, transportation, storage, refining, and distribution of petroleum products.

Finally, the pursuit of the best individual decisions by use of these or other means must always be conditioned by good judgment. L. Tornquist of the Cowles Foundation has observed that the decision maker who tries to make only the best decisions may be beaten by a decision maker who deliberately takes the risk of making some less-than-optimum decisions and as a result makes more and earlier decisions. The truly optimal strategy contains a large number of approximately optimal decisions. Occasionally some bad decisions have to be tolerated if the decision maker is to increase the speed of decision making, and by studying the consequences of bad decisions, it will be easier later on to find better decisions.

Mathematical analysis properly used can help in making good over-all decisions; improperly used, it may try to find a few "best" decisions at the cost of making many other "bad" decisions.

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* L. Tornquist, "The Best Can Be the Enemy of the Good," Cowles Commission Discussion Paper—Economics No. 2074.

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